10th Grade Assignment – Week #32

Announcements

- Prior to Tuesday's group meeting, you will need to print out three copies of the *Equitempered Scale Graph* (which is found at the end of this document).
- For Thursday's group assignment you will need a string instrument (if you have one). Those who do not have a string instrument should come to the meeting with a long jump rope (or fairly a thick rope with a length of about 4m (13 ft)), and have someone at the meeting with you to hold the other end of the rope.

Individual Work

- Finish anything from the "Group Assignment" that your group doesn't complete.

Group Assignment *for Tuesday* *Discovering the modern equitempered scale!*

- **Background**
  - Most of this is explained thoroughly in Lecture #1 (Monday).
  - This is the question we are aiming to answer:
    How many steps to the octave produce the best equitempered scale?
  - To answer the above question, you will do what I did in the lecture – where I created a 7-step equitempered scale and graphed the notes so that I could see how close the notes were to the basic Pythagorean Intervals (fifth, fourth, third and sixth).
- **Instructions**
  1) Everyone in the group will work with a different equitempered scale – each scale dividing the octave into a different number of equal steps. Assign each person in the group to one of these numbers of steps (in this order of importance): 8, 12, 19, 10, 13, 17, 22, 15, 31.
     Your task as a group is to see how many of these scales you can do. Everyone should do as many as they can. Help each other out!
  2) Now that you know the number of steps in your equitempered scale, you need to do the following:
     a) Calculate the value of "R", which tells you what to multiply by in order to get the frequency from one note to the next. For example, in the lecture, I used a 7-step scale, so I needed to solve this equation:
        $$440 \cdot R^7 = 880 \quad \rightarrow \quad R^7 = 2 \quad \rightarrow \quad R = \sqrt[7]{2} \approx 1.10409,$$
        which tells us that each step increases the frequency by 10.409%
        Considering the number of steps in your scale, what is your value of R ?
     b) You could now use this value of R to calculate all the notes in your scale (do you understand how to do this?). But I've made it easier for you! **All you need to do is open the spreadsheet I gave you** (10th Grade-Week #32-Frequency Calculator.xlsx) **and then change the number of steps from "7" to the number of steps you want.** The frequency of the notes in your scale are then automatically generated. You will only need to use the notes in the third octave – between 1760 and 3520.
     c) Using the *Equitempered Scale Graph* (which is found at the end of this document), record the number of steps at the top of the page, and then **very carefully and accurately** mark the steps so that they correspond to the notes in your scale. You only need to do this for the third octave – between 1760 and 3520.
  3) **15 minutes before the end of your group meeting, you need to discuss this question:**
     By looking at the tables you have made (however many your group completed),
     how many steps to the octave produce the best equitempered scale?
     (Remember that we want the notes in our scale to be very close (within 0.5% error) to the basic Pythagorean Intervals of: fifth (3:2), fourth (4:3), third (5:4), and sixth (5:3).)

Group Assignment *for Thursday*

- *Finding Harmonics.*
  Using your string instrument, complete the *Harmonic Worksheet* (found on the next page). Those without an instrument can do the calculations.
- *Spinning a Jump Rope.*
  Similar to what I did in the video, see if you can spin a rope to get three loops (as I did), or two loops, or four or five loops.
- *Calculating notes*. Assuming the modern (12-step) equitempered scale, answer these questions:
  4) What is the ratio of two neighboring notes (what is normally called a "half-step")?
  5) What is the ratio of a "fifth"? (Hint: remember that a fifth is 7 half-steps)
  6) What is the frequency of the note that is 7 octaves above A 27.5?
  7) What is the frequency of the note that is 12 fifths above A 27.5?
  8) Determine the number of half-steps needed to go from one note up to the next note:
     a) 98 Hz and 293.66 Hz?
     b) 659.26 Hz and 2093 Hz?
     c) 61.74 Hz and 3520 Hz?
  9) Determine the name of the note with each of the given frequencies. (Hint: Recall that the "A" notes have frequencies of 27.5, 55, 110, 220, 440, 880, 1760 and 3520 Hz.)
     a) 174.61
     b) 932.33
     c) 2217.5
     d) 246.94

Math and Music

Harmonic Worksheet

Here are the frequencies of a D string (using the modern equitempered scale):

- Violin or viola: 293.7 Hz.
- Cello or guitar: 146.8 Hz
- Bass: 73.4 Hz

Using one of the above instruments, demonstrate (on the D string) what is stated, and give the name of the resulting note.
(This is similar to what I did in the clip from my public *Math & Music* lecture.)

1. Pressing down at ½ the string length.
2. Touching at ½ the string length.
3. Pressing down at ¾ the string length.
4. Touching at ¾ the string length.
5. Pressing down at ¼ the string length.
6. Touching at ¼ the string length.
7. Pressing down at ⅔ the string length.
8. Touching at ⅔ the string length.
9. Pressing down at ⅓ the string length.
10. Touching at ⅓ the string length.
11. Pressing down at ⅘ the string length.
12. Touching at ⅘ the string length.
13. Pressing down at ⅗ the string length.
14. Touching at ⅗ the string length.
15. Pressing down at ⅖ the string length.
16. Touching at ⅖ the string length.
17. Pressing down at ⅕ the string length.
18. Touching at ⅕ the string length.

Equitempered Scale Graph

A Major – # Steps per Octave = ___