10th Grade Assignment – Week #31

Announcements

- String instruments will be needed again next week. For this week, you may find having a string instrument (or a piano) is helpful for a few of the exercises, but it is not at all necessary.

Individual Work

- Finish anything from the "Group Assignment" that your group doesn't complete.

Group Assignment

for Tuesday

- Do problems #1-3 from **Problem Set #3** but make sure you follow these instructions:
  - Before you do problem #1 (which requires you to fill out the table), you need to first read (and thoroughly understand!) the three paragraphs that are just before #1, as well as the "Notes" that appear just after #1 (and above the table).
  - With #2, it is best to give the ratios in whole number form.
  - With #3, you should make a new table (on a separate piece of paper), but simply start instead with the root note as D 297 and just do the first 8 notes (rows of the table). Be sure to also list the ratios of neighboring pairs of notes (as you dd for #2 with the C major scale).
  - **Important:** After you have completed both tables (for the C major and D major scales), you should discuss the following question: When comparing the results of these two tables, what is there that is surprising or concerning?
- *Going up by Fifths.* Starting at a C, how many fifths do you need to go up by until you get back to a C? Write down all of the notes you get along the way. (Hint: From **C** you go up a fifth to get to **G**. Then from G, you go a fifth to get to **D**, etc.)

for Thursday

- Do problems #1-10 from **Problem Set #2.**
- Do problems #4-9 from **Problem Set #3.** Also, make sure that you understand the "Harmonic Proportion" at the end of the problem set. (Note that "tone" means "whole step".)

Problem Set #2

Intervals between Notes

Early in the quest for the hidden mathematical laws behind the beauty of music, the Pythagoreans discovered a very important property – that the most pleasing musical intervals were those that had frequencies that could be reduced to simple whole number ratios. The simpler the ratio (e.g., 3:2), the more consonant, or harmonious, the interval. Intervals with more complex ratios (e.g., 256:243) would sound more dissonant to one's ear.

We can expand further what the Pythagoreans started by creating a complete musical scale based only on simple ratios. The table below shows the basic intervals.

Table of Intervals

Name of Interval	Ratio of Interval	Example
Second	9:8	C → D
Major 3rd	5:4	C → E
Fourth	4:3	D → G
Fifth	3:2	A → E
Major 6th	5:3	C → A
Major 7th	15:8	C → B
Octave	2:1	B → B'

For each resulting note, give the name and frequency (number of vibrations per second). Use the *Table of Intervals* as needed.

1) What is the frequency of the note that is a fifth up from A 440 Hz?

2) What is the frequency of the note that is a fourth up from A 440 Hz?

3) What is the frequency of the note that is a third up from F 352 Hz?

4) What is the frequency of the note that is an octave up from F 352 Hz?

5) What is the frequency of the note that is three octaves up from F 352 Hz?

6) What is the frequency of the note that is two octaves below F 352 Hz?

7) What is the frequency of the note that is 2 fifths above A 440 Hz?

8) What is the frequency of the note that is 2 fourths above A 440 Hz?

9) What is the frequency of the note that is a fifth and then a fourth above A 440 Hz?

10) What is the significance of the answer to the previous problem?

Problem Set #3

A Just Intonation Diatonic Scale

A basic law of acoustics is that *the frequency of a note is inversely proportional to the length of the string*. What does this mean? Loosely speaking, we all know that if we press our finger down at a place on a string (e.g., a violin or guitar) we get a higher pitch because we have made the string shorter.

Look at the string shown at the bottom of the page. (If this were a cello, then you would be bowing close to the right end of the string.) It is 60 cm long and is tuned to C 264 Hz. Where must you press in order to get a D? We see on the *Table of Intervals* that the ratio for a second interval (C→D) is 9:8. This tells us that if we press our finger at the place that makes the string $^8/_9$ as long ($60 \cdot {}^8/_9 = 53\frac{1}{3}$ cm) then the frequency becomes $^9/_8$ as much ($264 \cdot {}^9/_8 = 297$ Hz). That is exactly where you should press in order to get a D.

Also keep in mind that if a string is made half as long, then its frequency is doubled (i.e., it goes up by an octave).

1) Given that the string (at the bottom of the page) is 60 cm long and is tuned to C 264 Hz, fill in the table below. Give your answers as mixed numbers, not decimals.

Notes:

- The first eight notes form the notes in a C-major scale.
- After that, some of the note names are not in order – you have to think!
- There are no flats or sharps.
- Be sure that you do all calculations based on the interval that the note is from C 264 – otherwise you will encounter problems.

#	Name	Length	Frequency
1.	C	60 cm	264 Hz
2.	D	53⅓ cm	297 Hz
3.			
4.			
5.			
6.			
7.			
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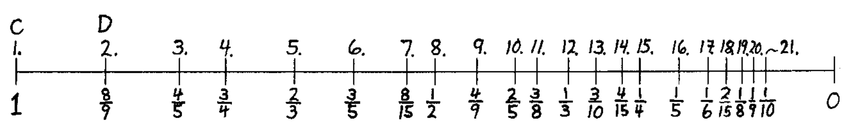

2) Using the table that you just created, calculate the ratios of the frequencies of all of the neighboring notes. What do these ratios show? What is it that is not ideal about this scale?

3) What would have happened if we had started the whole table from D 297, instead of C 264?

Three Special Intervals

We will now investigate three special intervals. The situations that produce each of these pairs of notes are described below.

In each case, you may choose a starting note to have any frequency that you wish. From this note you will be asked to create two new notes, one higher than the other. **Answer each problem by giving the ratio of the frequency of the higher note to the lower note**.

4) The higher note is a fifth above the starting note, and the lower note is a fourth above the starting note.

5) The higher note is produced by going up a fifth, two times, from the starting note, and the lower note is an octave above the starting note.

6) The higher note is an octave above the starting note, and the lower note is produced by going up a fourth, two times, from the starting note.

The Three Means & Music

The below problems demonstrate the importance of the three means in music.

7) a) Find the frequency and the name of the note that is the *arithmetic mean* of C 264 and C 528.

b) What is the ratio of the frequency of this new note to the starting note, C 264? What interval is it from the starting note?

8) a) Find the frequency and the name of the note that is the *harmonic mean* of C 264 and C 528.

b) What is the ratio of the frequency of this new note to the starting note, C 264? What interval is it from the starting note?

9) a) Find the frequency and the name of the note that is the *geometric mean* of C 264 and C 528.

b) What is the ratio of the frequency of this new note to the starting note, C 264? What interval is it from the starting note?

The Harmonic Proportion

All of this comes together in a very wonderful way, as shown in the diagram below.