10th Grade Assignment – Week #30

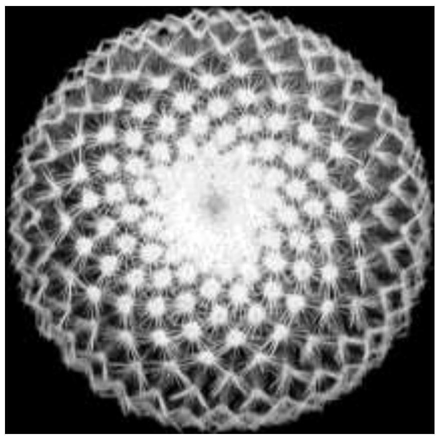
Announcements

- There will be no test for either of these last two units – *Sequences & Series* and *Math & Music.*
- Those who play a string instrument will need their instrument for Thursday's group work, and you'll need someone physically with you to assist with taking measurements.

Individual Work

- Finish anything from the "Group Assignment" that your group doesn't complete.
- **Note:** Do this problem before watching Lecture #2 (because I will go over it then).
  Counting Spirals. In Lecture #1, I showed you how spirals can appear in a pine cone. Spirals also can appear in sunflowers, cacti, and other plant forms. See if you can find something in nature that has spirals, and count the number of spirals. If you can't find anything outside, you can instead count the spirals in the photos found below. Keep in mind, apart from the spirals that first caught your attention, there is often another set of spirals going in a different direction. With each set of spirals, how many spirals are there?

Pine Cone



Cactus

Sunflower

- *Series.* Evaluate each of the following problems using one of the formulas from the Tuesday group assignment (also covered in Wednesday's Lecture #2). With all problems in sigma (∑) notation, list the first four terms, and then evaluate.

1) $\sum_{i=1}^{22} i$

2) $\sum_{i=1}^{71} 3i$

3) $\sum_{i=1}^{30} 7$

4) $\sum_{i=1}^{26} 4i - 6$

5) $\sum_{i=0}^{20} 3^i$

6) $\sum_{i=0}^{8} 7^i$

7) $\sum_{i=0}^{7} 10^i$

8) $\sum_{i=0}^{12} \left(\frac{1}{2}\right)^i$

9) $\sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i$

10) $\sum_{i=0}^{\infty} \left(\frac{1}{3}\right)^i$

11) $\sum_{i=0}^{\infty} \left(\frac{1}{10}\right)^i$

12) $\sum_{i=0}^{\infty} \left(\frac{3}{4}\right)^i$

13) $\sum_{i=0}^{\infty} \left(\frac{3}{8}\right)^i$

14) $\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots$

15) $1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots$

16) $\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \ldots$

17) $1 + \left(\frac{3}{7}\right) + \left(\frac{3}{7}\right)^2 + \left(\frac{3}{7}\right)^3 + \ldots$

18) $\left(\frac{3}{5}\right) + \left(\frac{3}{5}\right)^2 + \left(\frac{3}{5}\right)^3 + \ldots$

19) $\sum_{i=0}^{\infty} \left(\frac{9}{8}\right)^i$

Group Assignment

for Tuesday **Deriving Formulas for Series**

In the lectures, we have learned Gauss's Formula, which is: $\sum_{i=1}^{n} i = (^n/_2)(n+1)$

Use Gauss's Formula to evaluate each of these series:

1) $\sum_{i=1}^{36} i$

2) $\sum_{i=1}^{36} 5i$

3) $\sum_{i=1}^{36} 3$

4) $\sum_{i=1}^{36} 5i + 3$

Create a new formula that works for each of the below series. Note that a and b are each a constant. (E.g., for #4, a = 5 and b = 3).

5) $\sum_{i=1}^{n} ai$

6) $\sum_{i=1}^{n} i + b$

7) $\sum_{i=1}^{n} ai + b$

(Tuesday Group Assignment is continued on the next page →)

(Tuesday Group Assignment, cont.)

Discovering the Power Series Formula. (See how far you can get!)

I will now guide you through a process that leads to the creation of a completely different formula, which can help us to evaluate something like: $\sum_{i=0}^{13} 2^i$ where i (the counter) is in the exponent. (These problems are copied from Problem Set #6 of the *Sequences & Series* unit in the workbook.)

Multiply.

8) $(x-1)(x+1)$

9) $(x-1)(x^2+x+1)$

10) $(x-1)(x^3+x^2+x+1)$

11) $(x-1)(x^6+x^5+x^4+x^3+x^2+x+1)$

12) $(x-1)(x^{12}+x^{11}+\ldots+x+1)$

13) $(x-1)\sum_{i=0}^{6} x^i$

14) $(x-1)\sum_{i=0}^{22} x^i$

15) $(x-1)\sum_{i=0}^{n} x^i$

16) **Power Series Formula**. Use the previous problem to state a general formula for $\sum_{i=0}^{n} x^i$.

Evaluate each expression using the above formula:

17) $(1+3+3^2+3^3+3^4+3^5+3^6)$

18) $\sum_{i=0}^{6} 3^i$

19) $(1+2+2^2+\ldots+2^{18})$

20) $\sum_{i=0}^{18} 2^i$

21) $(1+10+10^2+\ldots+10^{13})$

<u>Infinite Series</u>

What happens if you add up infinitely many numbers? Most people would reply that you get an infinitely large result. But, what if the series had the special characteristic that each step got smaller?

22) Consider the following series: $\sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i$

a) Write down the terms of the series.
b) Evaluate the series. (Use a calculator, if necessary.)

Consider the following relation: $\mathbf{y = x^n}$.
Use a calculator to determine **y**, given that…

23) $n = 50$ and $x = 2$

24) $n = 50$ and $x = 1.8$

25) $n = 50$ and $x = 1.6$

26) $n = 50$ and $x = 1.2$

27) $n = 50$ and $x = 1.1$

28) $n = 50$ and $x = 1.05$

29) $n = 50$ and $x = 1.002$

30) $n = 50$ and $x = 1$

31) $n = 50$ and $x = 0.99$

32) $n = 50$ and $x = 0.95$

33) $n = 50$ and $x = 0.9$

34) $n = 50$ and $x = 0.7$

35) $n = 50$ and $x = 0.5$

36) $n = 50$ and $x = 0.3$

37) $n = 50$ and $x = 0.01$

38) What mathematical law is reflected in the above problems?

39) The *Power Series Formula* (that was found above) states:

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$

If $0 < x < 1$, then what can be said about the value of x^{n+1} as n approaches infinity?

40) **The Infinite Power Series Formula**

What does the *Power Series Formula* become when $0 < x < 1$ and n approaches infinity?

for Thursday (This marks the beginning of the *Math & Music* unit!)

Discovering the Ratios of the Basic Intervals of a Musical Scale

1) With a C major scale (the root note is C)…
   a) List all the notes in the scale.
   b) What is the note that is the "fifth" above the root note?
   c) What is the "fourth" (above the root)?
   d) What is the "major third" (above the root)?
   e) What is the "major sixth" (above the root)?

2) With an A major scale (the root note is A)…
   a) List all the notes in the scale.
   b) What is the "fifth" (above the root)?
   c) What is the "fourth" (above the root)?
   d) What is the "major third" (above the root)?
   e) What is the "major sixth" (above the root)?
   f) How many sharps and flats did you get?

3) **We will now do an experiment!** For those who have a string instrument (cello, violin, guitar, etc.) you will need someone (perhaps a parent or sibling) to help you take measurements. The other people in the work group (who don't have an instrument) can record all the data, and help with the calculations. In the end, you will fill out the two below tables for each person with an instrument. You should fill out the "A Major Sclae" first, and then fill out the "C Major Scale". Here are the instructions:

- *Note Name*. "Root" means the first note of the scale, and is the same as the name of the scale. The rest of the note names come from #1 and #2 above.
- *String Length*. This is where you will take measurements. For the "A Major Scale", you will play all of the notes on the A string. First measure the length (cm is best) of the "open" string, which is the full length of the vibrating portion of the string from the bridge up to where the string again connects/touches the upper part of the neck/scroll. This is then the "string length" for the root note. (A violin should be around 32.5cm, and a cello should be around 69cm.)
  Next comes the important part. For the "A Major Scale", on the A string, play the "fifth" (which is an E). Have someone else measure the length of the vibating portion of the string – from the bridge up to your finger. Record the measurement in the table. Repeat this for the rest of the notes (fourth, major third, etc.). All measurements should be made to the nearest millimeter. (Note: if your instrument doesn't have the string for that particular scale (e.g., a violin doesn't have a C string), then find the desired root note on another string (such that the length of the vibrating portion of the string is as long as possible) and then record that length as "string length" for the root note, and then proceed as normal for the rest of the notes.)
- *The Ratio of the Interval*. This means the ratio of the length of the whole A string to the length of the given note. For decimal form, you get this value by dividing the root string length by the string length of the given note. **Whole number form** (which is trickier) **is the main purpose of the whole experiment!** Because there is always a bit of error with measuring, you will need to guess a bit at what the whole numbers should be – and it should two single-digit numbers. For example, if the two string lengths were 70.3cm and 40.1cm, the ratios would be ≈1.75:1 (in decimal form) and 7:4 (in whole number form).
- *The Big Question!* After looking at all the tables, what do you think the perfect ratios really are?

A Major Scale

Interval	Note Name	String Length	Ratio of the Interval: Decimal Form	Ratio of the Interval: Whole # Form
Root				
Fifth				
Fourth				
Mj. Third				
Mj. Sixth				
Octave				

C Major Scale

Interval	Note Name	String Length	Ratio of the Interval: Decimal Form	Ratio of the Interval: Whole # Form
Root				
Fifth				
Fourth				
Mj. Third				
Mj. Sixth				
Octave				