10^{th} Grade Assignment – Week #29

Background

Important! Try to read everything below before your Tuesday group meeting.

- *Three Means*. In Lecture #1, I gave you formulas for the three means as:

Arithmetic Mean	Geometric Mean	Harmonic Mean
$A = \frac{X+Y}{2}$	$G = \sqrt{X \cdot Y}$	$H = \frac{2XY}{X+Y}$

- *Sequences*
  - A *sequence* is simply a list of numbers that follows a certain pattern.
  - An *Arithmetic Sequence* adds a "common difference" with each step.
    Example: The sequence X = 4, 7, 10, 13, 16, 19… has a common difference of 3.
  - A *Geometric Sequence* has a "common multiplier" (or ratio) with each step.
    Example: The sequence Y = 3, 6, 12, 24, 48, 96… has a common multiplier of 2.
- *Terminology.*
  - The initial term in a sequence is usually referred to as X_0 (although in some cases it is X_1).
  - Therefore, using the above sequence X, we can see that $X_0 = 4$, $X_1 = 7$, $X_2 = 10$, etc.
  - X_n means the n^{th} term in the sequence. X_{n-1} refers to the term before X_n. X_{n-2} refers to two terms before X_n.
- *Recursive Formulas*
  - A recursive formula gives instructions on how to calculate X_n based upon the value of the previous term or terms (X_{n-1} and/or X_{n-2}).
  - Therefore, we can say the following (make sure you really understand this!):
    - The recursive formula for sequence X (see above) is $X_n = X_{n-1} + 3$, which simply means: "The n^{th} term is found by taking the previous term and adding 3."
    - The recursive formula for sequence Y (see above) is $Y_n = 2 \cdot Y_{n-1}$, which simply means: "The n^{th} term is found by multiplying the previous term by 2."
  - Recursive formulas have limited usefulness. For example, to find the 100^{th} term, you have to first determine the first 99 terms. This is why we need a *General Formula*.
- *General Formulas*
  - A general formula gives instructions on how to calculate X_n based only upon the value of n and X_0 (and maybe X_1 as well).
  - The general formula for the sequence X (see above) is $X_n = 4 + 3n$. Let's check it!
    Example: What is X_5 (the 6^{th} term) in sequence X? The formula says $X_5 = 4 + 3 \cdot 5 = 19$
    What is X_{100} in sequence X? The formula says $X_{100} = 4 + 3 \cdot 100 = 304$
  - The general formula for the sequence Y (see above) is $Y_n = 3 \cdot 2^n$. Let's check it!
    Example: What is Y_5 in sequence Y? The formula says $Y_5 = 3 \cdot 2^5 = 96$
    What is X_{20} in sequence X? The formula says $Y_{20} = 3 \cdot 2^{20} = 3{,}145{,}728$

Individual Work

- Take the *Logarithms and Exponential Growth* quiz found at the end of this document.
- Finish anything from the "Group Assignment" that your group doesn't complete.

Group Assignment

for Tuesday

- Make sure everyone understands the "Background" listed above.

Answer these questions: (Answers are at the bottom of the next page.)

1) Find the three means (arithmetic, geometric, harmonic) for 5 and 45.
2) Find the three means (arithmetic, geometric, harmonic) for 23 and 27.
3) Given this sequence X = 5, 12, 19, 26…
   a) What kind of sequence is it?
   b) Give the next three terms and the previous three terms (before 5).
   c) Give the recursive formula.
   d) Give the general formula.
   e) Find X_{40}.
   f) Complete this sentence: "With any three consecutive terms in an *arithmetic sequence*, the middle term is always the _____ mean of the other two terms."
4) Given this sequence X = 80, 100, 125, 156.25…
   a) What kind of sequence is it?
   b) Give the next three terms and the previous three terms (before 80).
   c) Give the recursive formula.
   d) Give the general formula.
   e) Find X_{13}.
   f) Complete this sentence: "With any three consecutive terms in a *geometric sequence*, the middle term is always the __ ___ mean of the other two terms."
5) We will now create our first *Harmonic Sequence*.
   a) Complete this sentence: "With any three consecutive terms in a *harmonic sequence*, the middle term is always the _____ mean of the other two terms."

      The *harmonic mean* of 240 and 400 is 300. Confirm that this is true.

      Therefore, the first three terms in our harmonic sequence are 240, 300, 400.

   b) The *recursive formula for a harmonic sequence* is $x_n = \frac{x_{n-1} \cdot x_{n-2}}{2x_{n-2} - x_{n-1}}$

      (Deriving this formula from the formula for the harmonic mean (see above) is a challenge – try it, if you dare!)

      Use this recursive formula to find the next term (X_3) in the sequence.

   c) The *general formula for a harmonic sequence* is $x_n = \frac{x_0 \cdot x_1}{x_1 - n(x_1 - x_0)}$

      (Yes, deriving this formula is an even greater challenge!)

      Use this general formula to find X_4, X_5, X_{10}, X_{-1}, X_{-2}.

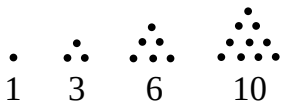
for Thursday

1) A Rabbit Sequence. In 1225, Leonardo de Pisa posed the following problem:
"Beginning with a single pair of baby rabbits, if every month each productive pair bears a new pair, how many pairs of rabbits will there be after each month?"
Give the number of pairs of rabbits at the end of each month for the first year.
(Assume that each pair mates at one month old, and the babies are born one month later. E.g., at the end of the first month, they mate, but there is still only 1 pair. At the end of the second month, a pair of babies are born, so there are 2 pairs in the field, etc.)

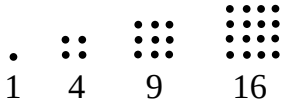
2) Polygonal Numbers

Long ago, the Pythagoreans investigated the properties of numbers that were geometric in nature. For each of the below sequences, give the next four terms in the sequence, and give both a recursive formula, and a general formula.
It will be more convenient to start each sequence with $X_1 = 1$.

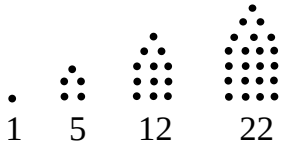
a) The *Triangular Numbers*



b) The *Square Numbers*



c) The *Pentagonal Numbers*



d) The *Hexagonal Numbers* (Figure it out!)

Logs & Exponential Growth Test

You may not use a calculator for Problems 1-11. You may instead use your Power and Base Tables.

(1 point each.)

1) $8^{1/3}$

2) 8^{-2}

3) $9^{-5/2}$

4) $\log_2 16$

5) $\log_9 3$

6) $\log_4 (\frac{1}{16})$

7) $\log_3 (-⅓)$

8) $\log_4 (½)$

9) $\log_9 (27)$

Solve each equation.
(3 points each.)

10) $5^{(3x+1)} = 125$

11) $11 + 4 \log_9(2x–5) = 5$

You <u>may</u> use a calculator for #12-16. (3 points each.)

12) The enrollment of a college is increasing by 9% per year. If the enrollment is currently 5600, then what will it be after 20 years if that growth rate continues?

13) A town's population grew from 15,327 to 18,802 over a 5-year period. What was the average annual growth rate?

14) An investment account is growing at 6% annually. How long does it take the account to triple?

15) At 2.8% APR, how long does it take an account to go from $800 to $1600 with monthly compounding?

16) Given an initial deposit of $8000, an APR of 3.4%, and a ten-year period of time that the money is invested in the account, calculate the ending balance given continuous compounding.