

10th Grade Assignment – Week #27

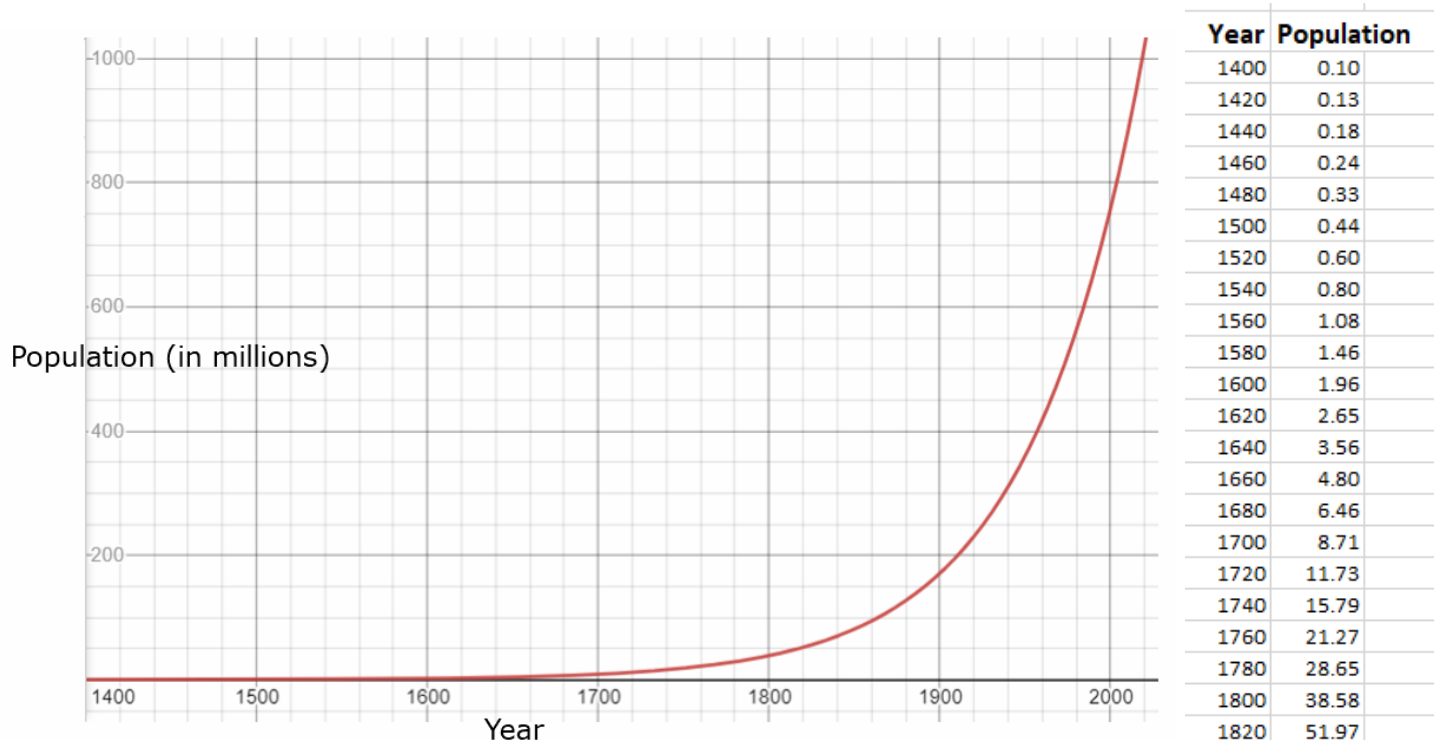
Individual Work

- Do as much as you can from the problems from **Problem Set #1** (*Exponential Growth* unit). Save problems #11-17 for Thursday's group work.
- Finish anything from the "Group Assignment" that your group doesn't complete.

Group Assignment

for Tuesday

- Do **Problem Set #2** (*Exponential Growth* unit), but follow these extra instructions:
 - For the country in #2, answer this question: (answers are below.)
 - 1) 1.5% annual growth, amounts to how much growth per decade?
 - 2) 1.5% annual growth, amounts to how much growth per 20-year period?
 - 3) 1.5% annual growth, amounts to how much growth per century?
 - 4) When was the country's population equal to 1 million?
 - For problem #2a, I did the graph for you (see below). Now answer the question in problem #2b ("When was the greatest growth?")
 - For problem #3, each person in your group should do a different graph. If you have more than three people in your group, then you can do additional time intervals. I have also done the calculations for you (below) to shorten your work. (Note the population is given in "millions", so that a population listed as 0.1 is really 100,000.)



for Thursday

- Do **Problem Set #3** (*Exponential Growth* unit).
- Do **Problem Set #1**: problems #11-17

Exponential Growth

Problem Set #1

Exponential Growth Formula

$$P = P_0(1+r)^t.$$

Note: All account problems assume no activity other than what is stated.

- 1) If you put \$5000 into an investment account for 30 years, what is the ending balance if the account has...
 - a) a 1% annual rate of return?
 - b) a 2% annual rate of return?
 - c) a 4% annual rate of return?
 - d) a 8% annual rate of return?
- 2) Sandy has \$7002.36 in an account that she started 8 years ago at 4.3% APR. How much money did she initially put into the account?
- 3) A town's population grew from 5324 to 8068 over a 20-year period. What was the average annual growth rate?
- 4) A town's population grew from 12,540 to 20,723. How long did this take if the average annual growth was 2.83%?
- 5) A bank account increases by 7% per year for 5 years. What percent increase is this over the 5-year period?
- 6) Bill was told that if he put his money into the local bank's savings account then his money would increase by exactly 20% after 5 years. Bill thought to himself, "Then the APR must be exactly 4%, which is the 20% divided evenly over the five

years."

But this is wrong.

- a) Why is Bill wrong?
 - b) What is the correct APR?
- 7) The enrollment of a college is increasing by 15% per year. If the enrollment is currently about 3000, then what will it be after 18 years if that growth rate continues?
 - 8) Kim's investment doubled over a ten-year period. What was her average annual rate of return during this period?
 - 9) A city's population is growing at 4% annually. How long does it take the population...
 - a) to double?
 - b) to triple?
 - c) to increase by 62½%?
 - d) to go from 80,000 to 130,000?
 - 10) John deposits \$440 in an investment account and then after 7 years the balance is exactly \$880. Find the average annual rate of return on his investment, and show the balance after each year.

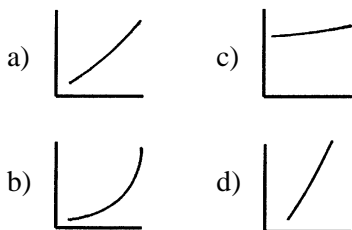
For each problem below, describe what happens to y as x gets larger and larger.

- 11) $y = 2^x$
- 12) $y = (1.1)^x$
- 13) $y = 1^x$
- 14) $y = (0.8)^x$
- 15) $y = (0.5)^x$
- 16) $y = (0.5)^{-x}$
- 17) $y = (1.25)^{-x}$

Problem Set #2

Exponential Growth

- 1) Which of the below graphs has the greatest growth rate?



- 2) a) Assume that for the last 600 years a country has been growing at a constant rate of 1.5% annually. On a piece of graph paper, carefully graph the population of the country from the year 1400 to 2000, given that the population in 1400 was 100,000.

Notes:

- The standard is that the horizontal axis should represent time, and the vertical axis should represent population.
 - Plotting between 10 and 15 points is sufficient.
- b) Look at the graph and estimate when the greatest rate of growth was.
- 3) Now, on three separate pieces of graph paper, graph the population of the same country during these three time intervals: 1400-1500, 1650-1750, and 1900-2000. (Be sure that you adjust the

starting and ending values of the vertical and horizontal axes so that each graph runs from the bottom left to the top right of the page.)

What conclusions can be reached after looking at these three graphs?

- 4) Now look again at question #1. Is there anything that you can say differently now?
- 5) What conclusions or general statements can you make about what you have learned on this problem set thus far?
- 6) How long does it take a country to have its population increase by a factor of 7 if it has a 1.5% annual growth rate?
- 7) If a country has a 1.5% annual growth rate, by what percentage does it grow every decade? Every century?
- 8) In 1982 Indonesia had a population of 157 million, and in 1996 it was 206 million. What was the average annual growth rate over that period?
- 9) Make a graph using the below table, and then calculate the average annual growth rate of San Francisco...
- from 1860-1870
 - from 1920-1930
 - from 1860-1900
 - from 1930-2000

Population of San Francisco (in thousands)									
Year	1860	1870	1880	1890	1900	1910	1920	1930	2000
Population	57	149	234	299	343	417	507	634	777

Problem Set #3

Types of Compounded Interest

If we say that interest is *compounded annually*, then it means that the amount of interest is calculated annually, and then the balance is adjusted accordingly. *Monthly compounding* means the interest is calculated (and the balance adjusted) every month.

If the number of compoundings per year is n , then the effective interest is r/n and the total number of compoundings is $n \cdot t$ (where r is the APR and t is years), which leads us to the *Compound Interest Formula*:

$$P = P_0 \left(1 + \frac{r}{n}\right)^{nt}$$

- 1) Find the balance after 10 years of an account with interest compounded monthly at 1.8% APR with an initial deposit of \$1360.
- 2) At 1.8% APR, how long does it take an account to go
 - a) from \$400 to \$1200 with annual compounding?
 - b) from \$800 to \$2400 with annual compounding?
 - c) from \$800 to \$2400 with quarterly compounding?
 - d) from \$800 to \$2400 with monthly compounding?
- 3) (For the following problems, use a calculator on a computer to get increased accuracy.) Given an initial deposit of \$700, an APR of 2.4%, and a five-year period of time that the money is invested in the account,

calculate the ending balance given...

- a) annual compounding
 - b) quarterly (four times per year) compounding
 - c) monthly compounding
 - d) daily compounding
 - e) hourly compounding
 - f) compounding every minute
 - g) compounding every second
 - h) continuous compounding
- 4) Ten years ago, Jack purchased a bike on his credit card for \$620, but has not made any payments on this debt. How much does he now owe if the credit card company charges 12% APR compounded monthly?
 - 5) If you go back five generations then you can see that you have 32 great-great-great-grand parents (which can be written as $\text{great}^3\text{-grand}$ parents). Now go back a total of forty generations.
 - a) Given that one generation is about 25 years, how many years is forty generations?
 - b) Going back forty generations, how many $\text{great}^{38}\text{-grand}$ parents do you have?
 - c) What can be said about your above answer?