10th Grade Assignment – Week #24

Individual Work

Finish anything from the "Group Assignment" that your group doesn't complete.

Group Assignment

for Tuesday

- **What we know so far.** Make sure everyone understands:
  - We used the 30-60-90 special triangle to determine the values of the sin 30° and cos 30°.
  - From there, we worked upwards on the table to calculate the sine and cosine of half that angle by using **Ptolemy's Half Angle Formula** and the **Pythagorean Trigonometric Identity** (both shown below). The resulting *Table of Half Angles* is found at the right.

Ptolemy's Half-Angle Formula

$$\sin(\tfrac{1}{2}\alpha) = \sqrt{\tfrac{1}{2} - \tfrac{1}{2}\cos\alpha}$$

Pythagorean Trigonometric Identity

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

  - How did we then get the **sin 1°** from our calculation of **sin** $\left(\frac{15}{16}\right)$? Well, that was rather tricky, but the *Ratio of Sines Table* and the law (below, in the box) helped us. From this, we were able to say that **sin 1°** $\approx \frac{16}{15}\sin\left(\frac{15}{16}\right) \approx \frac{16}{15} \cdot (0.016355) \approx$ **0.0174525**.
  - Then we used the **Pythagorean Trigonometric Identity** to calculate the **cos 1°**.
- Do this now: Starting with the values for sin 1° and cos 1° shown on the above table, calculate the values for:
  sin ½° cos ½° sin ¼° cos ¼°
- New Formulas: Ptolemy then created his sum and difference formulas (see below) in order to fill out the rest of his table. Go ahead now and fill out the rest of the (partial) table, shown on the right. The below example may help get you started.

Ptolemy's Sum and Difference Formulas

$$\sin(\beta - \alpha) = \sin(\beta)\cos(\alpha) - \sin(\alpha)\cos(\beta)$$
$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

Example: Since we know the trig values for 7½° and 1°, we can use the above formula to find sin (6½°).
$\sin(\beta - \alpha) = \sin(\beta)\cos(\alpha) - \sin(\alpha)\cos(\beta)$. **β = 7½, α = 1**
$\sin(7\tfrac{1}{2}° - 1°) = \sin(7\tfrac{1}{2}°)\cos(1°) - \sin(1°)\cos(7\tfrac{1}{2}°)$
$\sin(6\tfrac{1}{2}°) \approx (0.130526)(0.999848) - (0.0174524)(0.991445)$
$\sin(6\tfrac{1}{2}°) \approx 0.113203$

for Thursday: Do **Problem Set #8**, problems #2-6. Use the full *Table of Sines and Cosines* (below) instead of your calculator.

Table of Half Angles

α	sin(α)	cos(α)
1°	0.0174525	0.999848
15/16°	0.016355	0.99986625
1 7/8°	0.032714	0.999465
3 3/4°	0.0654031	0.997859
7 1/2°	0.130526	0.991445
15°	0.2588194	0.9659257
30°	0.5	0.866025

Ratio of Sines Table

y	x	sin y	sin x	sin y / sin x
120°	60°	0.866	0.866	1
90°	45°	1	0.707	1.414
60°	30°	0.866	0.5	1.73
2°	1°	0.1745	0.01746	1.99970

For very small angles,
y : x ≈ sin y : sin x

Partial Table of Sines and Cosines

α	sin(α)	cos(α)
0°		
(¼°	0.00436331	0.999991)
½°	0.00872654	0.999962
1°	0.0174524	0.999848
1½°		
2°		
2½°		
3°		
3½°		
(3¾°	0.0654031	0.997859)
4°		
4½°		
5°		
5½°		
6°		
6½°		
7°		
7½°	0.130526	0.991445
8°		

Table of Sines and Cosines

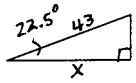
α	*sin*(α)	*cos*(α)	α	*sin*(α)	*cos*(α)
0.0°	0.00000	1.00000	22.5°	0.38268	0.92388
0.5°	0.00873	0.99996	23.0°	0.39073	0.92050
1.0°	0.01745	0.99985	23.5°	0.39875	0.91706
1.5°	0.02618	0.99966	24.0°	0.40674	0.91355
2.0°	0.03490	0.99939	24.5°	0.41469	0.90996
2.5°	0.04362	0.99905	25.0°	0.42262	0.90631
3.0°	0.05234	0.99863	25.5°	0.43051	0.90259
3.5°	0.06105	0.99813	26.0°	0.43837	0.89879
4.0°	0.06976	0.99756	26.5°	0.44620	0.89493
4.5°	0.07846	0.99692	27.0°	0.45399	0.89101
5.0°	0.08716	0.99619	27.5°	0.46175	0.88701
5.5°	0.09585	0.99540	28.0°	0.46947	0.88295
6.0°	0.10453	0.99452	28.5°	0.47716	0.87882
6.5°	0.11320	0.99357	29.0°	0.48481	0.87462
7.0°	0.12187	0.99255	29.5°	0.49242	0.87036
7.5°	0.13053	0.99144	30.0°	0.50000	0.86603
8.0°	0.13917	0.99027	30.5°	0.50754	0.86163
8.5°	0.14781	0.98902	31.0°	0.51504	0.85717
9.0°	0.15643	0.98769	31.5°	0.52250	0.85264
9.5°	0.16505	0.98629	32.0°	0.52992	0.84805
10.0°	0.17365	0.98481	32.5°	0.53730	0.84339
10.5°	0.18224	0.98325	33.0°	0.54464	0.83867
11.0°	0.19081	0.98163	33.5°	0.55194	0.83389
11.5°	0.19937	0.97992	34.0°	0.55919	0.82904
12.0°	0.20791	0.97815	34.5°	0.56641	0.82413
12.5°	0.21644	0.97630	35.0°	0.57358	0.81915
13.0°	0.22495	0.97437	35.5°	0.58070	0.81412
13.5°	0.23345	0.97237	36.0°	0.58779	0.80902
14.0°	0.24192	0.97030	36.5°	0.59482	0.80386
14.5°	0.25038	0.96815	37.0°	0.60182	0.79864
15.0°	0.25882	0.96593	37.5°	0.60876	0.79335
15.5°	0.26724	0.96363	38.0°	0.61566	0.78801
16.0°	0.27564	0.96126	38.5°	0.62251	0.78261
16.5°	0.28402	0.95882	39.0°	0.62932	0.77715
17.0°	0.29237	0.95630	39.5°	0.63608	0.77162
17.5°	0.30071	0.95372	40.0°	0.64279	0.76604
18.0°	0.30902	0.95106	40.5°	0.64945	0.76041
18.5°	0.31730	0.94832	41.0°	0.65606	0.75471
19.0°	0.32557	0.94552	41.5°	0.66262	0.74896
19.5°	0.33381	0.94264	42.0°	0.66913	0.74314
20.0°	0.34202	0.93969	42.5°	0.67559	0.73728
20.5°	0.35021	0.93667	43.0°	0.68200	0.73135
21.0°	0.35837	0.93358	43.5°	0.68835	0.72537
21.5°	0.36650	0.93042	44.0°	0.69466	0.71934
22.0°	0.37461	0.92718	44.5°	0.70091	0.71325

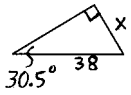
α	*sin*(α)	*cos*(α)
45.0°	0.70711	0.70711
45.5°	0.71325	0.70091
46.0°	0.71934	0.69466
46.5°	0.72537	0.68835
47.0°	0.73135	0.68200
47.5°	0.73728	0.67559
48.0°	0.74314	0.66913
48.5°	0.74896	0.66262
49.0°	0.75471	0.65606
49.5°	0.76041	0.64945
50.0°	0.76604	0.64279
50.5°	0.77162	0.63608
51.0°	0.77715	0.62932
51.5°	0.78261	0.62251
52.0°	0.78801	0.61566
52.5°	0.79335	0.60876
53.0°	0.79864	0.60182
53.5°	0.80386	0.59482
54.0°	0.80902	0.58779
54.5°	0.81412	0.58070
55.0°	0.81915	0.57358
55.5°	0.82413	0.56641
56.0°	0.82904	0.55919
56.5°	0.83389	0.55194
57.0°	0.83867	0.54464
57.5°	0.84339	0.53730
58.0°	0.84805	0.52992
58.5°	0.85264	0.52250
59.0°	0.85717	0.51504
59.5°	0.86163	0.50754
60.0°	0.86603	0.50000
60.5°	0.87036	0.49242
61.0°	0.87462	0.48481
61.5°	0.87882	0.47716
62.0°	0.88295	0.46947
62.5°	0.88701	0.46175
63.0°	0.89101	0.45399
63.5°	0.89493	0.44620
64.0°	0.89879	0.43837
64.5°	0.90259	0.43051
65.0°	0.90631	0.42262
65.5°	0.90996	0.41469
66.0°	0.91355	0.40674
66.5°	0.91706	0.39875
67.0°	0.92050	0.39073
67.5°	0.92388	0.38268
68.0°	0.92718	0.37461
68.5°	0.93042	0.36650
69.0°	0.93358	0.35837
69.5°	0.93667	0.35021
70.0°	0.93969	0.34202
70.5°	0.94264	0.33381
71.0°	0.94552	0.32557
71.5°	0.94832	0.31730
72.0°	0.95106	0.30902
72.5°	0.95372	0.30071
73.0°	0.95630	0.29237
73.5°	0.95882	0.28402
74.0°	0.96126	0.27564
74.5°	0.96363	0.26724
75.0°	0.96593	0.25882
75.5°	0.96815	0.25038
76.0°	0.97030	0.24192
76.5°	0.97237	0.23345
77.0°	0.97437	0.22495
77.5°	0.97630	0.21644
78.0°	0.97815	0.20791
78.5°	0.97992	0.19937
79.0°	0.98163	0.19081
79.5°	0.98325	0.18224
80.0°	0.98481	0.17365
80.5°	0.98629	0.16505
81.0°	0.98769	0.15643
81.5°	0.98902	0.14781
82.0°	0.99027	0.13917
82.5°	0.99144	0.13053
83.0°	0.99255	0.12187
83.5°	0.99357	0.11320
84.0°	0.99452	0.10453
84.5°	0.99540	0.09585
85.0°	0.99619	0.08716
85.5°	0.99692	0.07846
86.0°	0.99756	0.06976
86.5°	0.99813	0.06105
87.0°	0.99863	0.05234
87.5°	0.99905	0.04362
88.0°	0.99939	0.03490
88.5°	0.99966	0.02618
89.0°	0.99985	0.01745
89.5°	0.99996	0.00873
90.0°	1.00000	0.00000

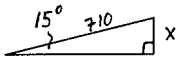
Problem Set #8

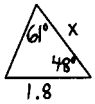
1) Write down every trigonometric formula and identity that you have seen in this unit, and state why it would be useful.

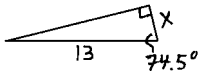
2) Find the variable indicated.

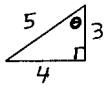
a) 

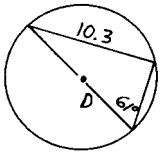
b) 

c) 

d) 

e) 

f) 

g) 

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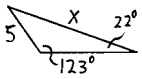
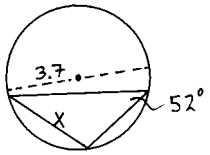
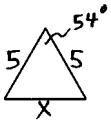
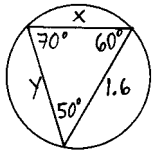
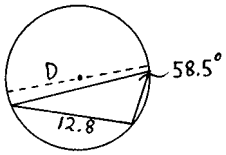
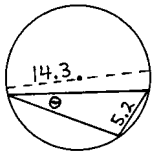
i) 

j) 

k) 

l) 

m) 

n) 

Word Problems with Trig

3) An 18-foot ladder is placed on level ground up against a building. If the angle of inclination of the ladder is 72°, then how far away from the side of the building is the foot of the ladder?

4) Jill is standing 200 feet from the base of a tall building. If the angle of inclination from where she is standing to the top of the building is 68°, then how tall is the building?

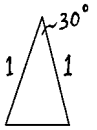
5) A rhombic dodecahedron is a polyhedron with 12 congruent faces that are rhombuses.

Within each rhombus, the ratio of the length of an edge to the shorter diagonal is $\sqrt{3}$:2.

a) Find the measures of the angles within the rhombus.

b) Find the ratio of the length of the longer diagonal to the shorter diagonal in the rhombus.

6) Find the area of this triangle. (This problem also appeared in the *Mensuration* unit.)



Calculating the Height of a Mountain

In 1852, Radhanath Sikdar discovered that "Peak XV" was the highest in the world. The General Surveyor of India, Captain Andrew Waugh, then led an effort to calculate the height of this mountain as accurately as possible. Several readings, all taken from the plains of India, were averaged. In 1856 it was announced that the height of this mountain, which Waugh named after his predecessor, Sir George Everest, was 29,002 feet. This measurement was within 35 feet!

How could this have been done?

[In place of the following exercise, it may be best to find a nearby mountain, hill, or tall building and try to calculate its height in a similar fashion. This would require the use of a sextant in order to measure the angle of inclination.]

7) Mount Katherine rises up from the western shore of Moose Lake, which has an elevation of 1723 feet. With a sextant, Jack measured that the angle of inclination from the western end of the lake to the peak of the mountain was 10°43' (which is $10\frac{43}{60}$), and from the eastern shore it was 4°16'. The distance between these two locations (where the angle of inclination measurements were taken) is 6.32 miles.

(From the peak of the mountain the whole of the lake can be seen. The mountain's peak and the two points of measurement all fall in a straight line on a map.

You may use the trig buttons on your calculator.)

a) What is the elevation of Mount Katherine?

b) How far would a map show that the distance is from the western shore to the peak, as a straight (horizontal) line?