10^{th} Grade Assignment – Week #23

Note: During the rest of this unit, there will be times where you can use a calculator (mostly just to make multiplication and division easier), but you should *not* use the trig buttons (sin and cos) as this would spoil the fun, and you wouldn't learn as much.

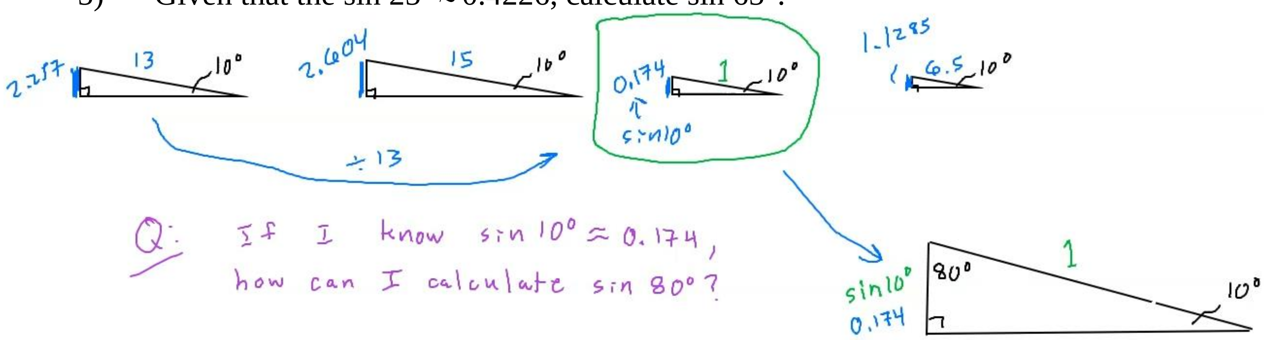
Individual Work

- Finish anything from the "Group Assignment" that your group doesn't complete.

Group Assignment

for Tuesday

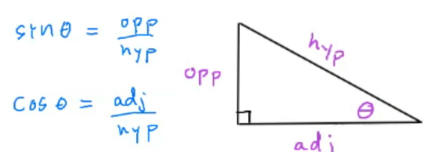
- Part of the first lecture this week was quite confusing, but very important. Part of our goal with group work today is to sort out some of this confusion.
  There is a lot here, so try to be efficient with your time!
  This was a big question: ***If I know sin 10° ≈ 0.174, how can I calculate the sin 80°?***
  The below questions lead us to an answer to the above question:
  1) Below, there is a line of four triangles. What do all of these triangles have in common?
  2) How did we use the first triangle to calculate the short sides of the other three triangles?
  3) Why is it special when the hypotenuse of the triangle is equal to 1?
  4) How did I use this to then calculate sin 80°? Do that (again) now.
  5) Given that the sin 25° ≈ 0.4226, calculate sin 65°.



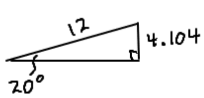
---

6) At the end of the lecture, I introduced the "Complementary Sine", what is called **Cosine**, and is abbreviated as **Cos**.
   Look at the **Basic Trig Triangle** and answer these questions:
   What question should we ask ourselves for **Sin**?
   What question should we ask ourselves for **Cos**?

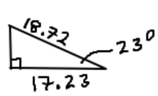
The Basic Trig Triangle



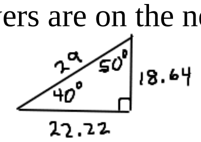
7) Calculate Sin and Cos, as indicated. (Answers are on the next page.)

a) 

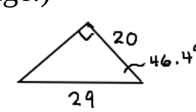
sin 20° =

b) 

cos 23° =

c) 

sin 40° =
sin 50° =
cos 40° =
cos 50° =

d) 

cos 46.4° =
sin 46.4° =
cos 43.6° =
sin 43.6° =

Tuesday's Group Work is continued on the next page!

Special Triangles

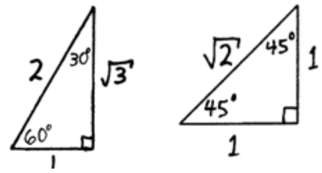


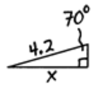
Table of Sines and Cosines

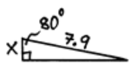
α	sin(α)	cos(α)
0°		
10°		
20°		
30°		
40°		
45°		
50°		
60°		
70°		
80°		
90°		

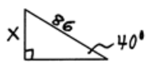
Tuesday Group Assignment (Continued)

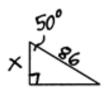
8) Fill out the *Table of Sines and Cosines*, shown below.
Some of this is a valuable review from last week.
The *Special Triangles* may be helpful.
Check your answers to be sure they are accurate.

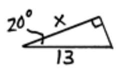
9) Use the *Table of Sines and Cosines* that you have just created in order to solve the following problems.
(This may be completed instead on Thursday.)

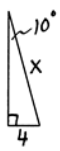
a) 

b) 

c) 

d) 

e) 

f) 

for Thursday

- Finish the problems from #9, above.
- Explain to one another what each of these identities mean, and how we know it to be true:

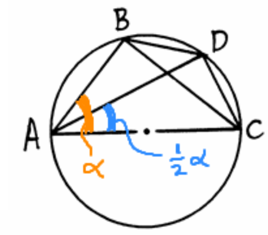
  $sin(180° - \alpha) = sin(\alpha)$ $\quad$ $sin(90° - \alpha) = cos(\alpha)$ $\quad$ $sin^2\alpha + cos^2\alpha = 1$

- *Challenge!* Together, as a group, work through the *Proof of the Half-Angle Formula*, which is on the next page. You can try to justify each of the steps.

Proof of the Half-Angle Formula

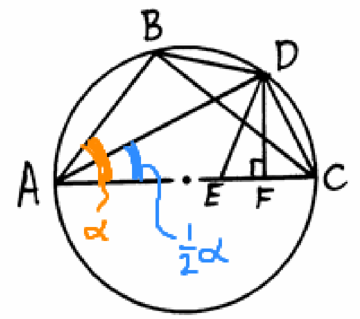
This proof is largely modeled after Ptolemy's proof of his half-angle formula.

The purpose of the proof



- We are given angle α (in the drawing, $\alpha = \angle BAC$), and we start off knowing the value of *sin*(α) and *cos*(α). Also, AD bisects α. Our goal is to find the *sine* of half of α. This means that in the drawing we aim to find the *sine* of ∠DAC.

The Proof



1) Place F on AC such that $DF \perp AC$.
   Place E on AC such that $AB \cong AE$.
2) $\Delta BAD \cong \Delta EAD$; $BD \cong DE$; $BD \cong CD$;
   $DE \cong CD$
3) ΔCDE is isosceles; $EF \cong CF$
4) $AC = AE + EF + CF$;
   $AC = AB + CF + CF$;
   $CF = \frac{1}{2}(AC - AB)$
5) $CD^2 = CF^2 + DF^2$;
   $DF^2 = AF \cdot CF$;
   $CD^2 = CF^2 + AF \cdot CF$;
   $CD^2 = CF(CF+AF)$;
   $CD^2 = CF \cdot AC$;
   $CD^2 = AC(\frac{1}{2}(AC - AB))$
   $CD^2 = \frac{1}{2}AC(AC - AB)$
6) $sin(\alpha/2) = \frac{CD}{AC}$; $CD = AC\ sin(\alpha/2)$;
   $cos(\alpha) = \frac{AB}{AC}$; $AB = AC\ cos(\alpha)$
7) Combining steps 5 and 6 gives us:
   $[AC\ sin(\alpha/2)]^2 = \frac{1}{2}AC(AC - AC\ cos(\alpha))$;
   $AC^2\ [sin(\alpha/2)]^2 = \frac{1}{2}AC^2 - \frac{1}{2}AC^2\ cos(\alpha)$;
   $[sin(\alpha/2)]^2 = \frac{1}{2} - \frac{1}{2}\ cos(\alpha)$;

$$\boxed{sin(\alpha/2) = \sqrt{\tfrac{1}{2} - \tfrac{1}{2}\ cos(\alpha)}}$$