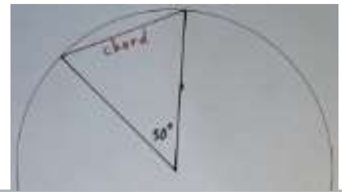
10th Grade Assignment – Week #22



D	R	θ	Chord	Ch : D
20	10	50°	8.2	0.41 : 1
12	6	50°	4.8	0.40 : 1
18	9	50°	7.6	0.42 : 1
16	8	50°	6.8	0.425 : 1

Group Assignment

for Tuesday

In Lecture #1, we measured the chord lengths of a few different angles, and then calculated the ratio of Ch:D (the chord to the diameter). Here, on the right, we see a table from that lecture which shows the results from four different sized circles, but in each case the central angle was 50°. Therefore, the ratio was always the same (although there was some error involved in the measurement). I said in the lecture that once we learn trigonometry, we can know that the correct ratio for this 50° angle should be approximately 0.422618.

1. Let's do one more measurement exercise, just for practice. Using a protractor, compass, and ruler, each person in the group should make an 76° central angle, but using a different sized circle. What value do you get for the ratio of Ch : D? Did you all get the same answer?
2. Ptolemy created a table of these values for angles ranging from 0° to 180°, with half-degree increments – for a total of 360 values. The amazing thing is that he did not measure at all – he instead used his thinking and precise calculations. How did he do this??? We'll see!
   But for now, we will look at the first few rows of his table (in the original Greek!) and try to make sense of it.
   But first, you need to know that Ptolemy did not write his values as ratios. Instead, he assumed that the circle always had a diameter of 120 (which he felt was a convenient number), and he only calculated the length of the (subtended) chord. Therefore, Ptolemy's table has 360 rows, but only two columns – one for the angle measure, and one for the chord length.
   Now, your task is to do **Problem Set #2**. You will need to carefully read (and understand!) the text at the beginning, especially the two paragraphs that start with the words "You may recall…" This will help you to understand how I filled in the last row of the table at the end of the problem set. You then need to translate from Greek and fill in the rest of the table. Have fun!
3. (If you still have time…) Determine how you can calculate the ratios of Ch : D (as we did in the lecture and with the table at the top of this page) for the following angles: 60°, 90°, and 120°. But the trick is to do this without measuring – just using your thinking and some math skill!

for Thursday

- Get as far as you can on **Problem Set #4**, but here are two things I'd like to add:
  - Discuss this question: Why are the answers for #1b and #1c the same?
  - Use the below values (from the lecture) to get started with the table in #3. You can then calculate the rest (without measuring!).

$$\sin 10° \approx 0.1736$$
$$\sin 50° \approx 0.7660$$
$$\sin 70° \approx 0.9397$$

Individual Work

- Do any more preparation needed for the *Algebra Review* test, and then take it (it is found at the end of this document).

Problem Set #2

Ptolemy's Chord Table

Klaudios Ptolemaios, known as Ptolemy, lived in Alexandria around 150A.D. His greatest book's title is translated as *The Mathematical Collection*, but it is usually referred to as *The Almagest*. Sections of this book are consider fundamental to the development of trigonometry. Most impressively, he constructed a *Table of Chords*, which listed the lengths of all of the chords, in a circle with a diameter of 120, with arcs up to 180° by increments of ½°. A small section of that table is shown on the next page. The headings of the left and right columns translate to "Arcs" and "Chords".

But to understand it we will need to first get a grasp of the ancient Greek number system. The Greek number system was non-positional. For example, in our number system, the symbol "7" has a very different meaning depending on the place in which it falls (e.g., 70 and 7000). The below chart will help us to translate the Greek numbers.

Notice that the Greek letters are used to represent numbers. Capital or small letters can be used to represent the same number. The only other symbol we will need to know is ∠' which is ½. Here are some examples:

κβ is 22; λζ is 37; τζ is 307; οα∠' is 71½.

You may recall that the Greeks didn't use decimals. Ptolemy actually writes his fractions in terms of minutes ($^1/_{60}$) and seconds ($^1/_{3600}$). For example, the bottom right entry of his table (given on the next page) is "ζ ν νδ", which we translate to "7, 50, 54", and means

$7 + \frac{50}{60} + \frac{54}{3600} \approx 7.8483$

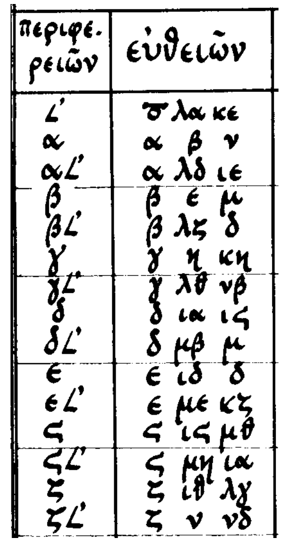
Taken together with the last entry of the left column (which reads 7½°), it says that an arc of 7½° has a chord of length 7.8483 – again assuming a circle with a diameter equal to 120.

Α	α	1	Ι	ι	10	Ρ	ρ	100	,α	1000
Β	β	2	Κ	κ	20	Σ	σ	200	,β	2000
Γ	γ	3	Λ	λ	30	Τ	τ	300	,γ	3000
Δ	δ	4	Μ	μ	40	Υ	υ	400	,δ	4000
Ε	ε	5	Ν	ν	50	Φ	φ	500	,ε	5000
Ϛ	ϛ	6	Ξ	ξ	60	Χ	χ	600	,ϛ	6000
Ζ	ζ	7	Ο	ο	70	Ψ	ψ	700	,ζ	7000
Η	η	8	Π	π	80	Ω	ω	800	,η	8000
Θ	θ	9	Ϙ	ϙ	90	Ϡ	ϡ	900	,θ	9000

ō, ō ϖc: 0

Given the excerpt from Ptolemy's Table of Chords, as shown on the right, and the table of the Greek numbers (on the previous page), fill in the below table. The last one of the 15 entries has been done for you.

	Arcs	Chords	Decimal
1)			
2)			
3)			
4)			
5)			
6)			
7)			
8)			
9)			
10)			
11)			
12)			
13)			
14)			
15)	7½°	7, 50, 54	7.8483



περιφερειῶν	εὐθειῶν
∠'	ō λα κε
α	α β ν
α∠'	α λδ ιε
β	β ε μ
β∠'	β λζ δ
γ	γ η κη
γ∠'	γ λθ νβ
δ	δ ια ιϛ
δ∠'	δ μβ μ
ε	ε ιδ δ
ε∠'	ε με κζ
ϛ	ϛ ιϛ μθ
ϛ∠'	ϛ μη ια
ζ	ζ ιθ λγ
ζ∠'	ζ ν νδ

We have been left to wonder how Ptolemy did these calculations. We will be following Ptolemy's logic and ideas as we construct our own Table, but not a *Table of Chords* as he did. Ours will be a *Table of Sines and Cosines*, which are the key elements of modern trigonometry.

Are you curious? Hopefully so.

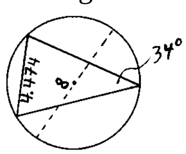
Problem Set #4

At this point, we have learned the following about *sine* (make sure you understand it!):

- That *sine* involves the relationship between an inscribed angle and the chord that subtends it.
- The meaning of a *sine* expression. For example, $sin(23°) \approx 0.3907$ tells us that if the inscribed angle is 23°, then the chord will be 0.3907 times as long as the circle's diameter.
- $sin(\alpha) = \frac{\text{chord}}{\text{diameter}}$
- $sin(180–\alpha) = sin(\alpha)$. (Why is this so?)

1) Calculate the *sine* of the given angle.

a)



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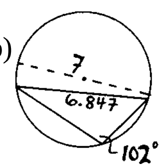

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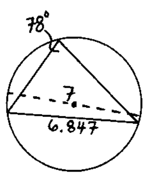

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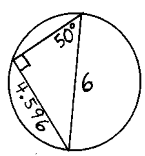
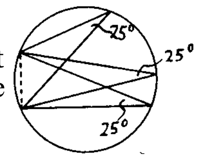


2) a) What is it that makes the last problem (#1d) different from the others?

b) Use #1d to calculate $sin(40°)$

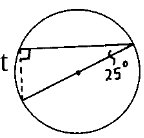
The Advantages of the Right Triangle

In order to form a picture of $sin(25°)$, it doesn't matter where the vertex of the angle sits.



In any case $sin(25°) \approx 0.423$, which means that the chord (shown as a dotted line) is about 0.423 times as long as the diameter of the circle.

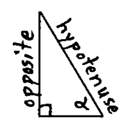
If instead we choose the vertex of the angle to be positioned so that a right triangle is formed, then it becomes much easier.



First of all, we notice that the hypotenuse of the triangle *is* a diameter. Now we can ignore the circle! Instead of sine being about the relationship of the chord to the diameter of the circle, in the case of a right triangle, it is about the relationship of the side opposite the angle to the hypotenuse of the triangle.

Therefore, reworking what we said earlier, we get the following for the case of a right triangle:

- $sin(\alpha)$ essentially answers the question: "The opposite side is what proportion of the hypotenuse?"
- $sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}}$

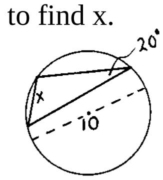


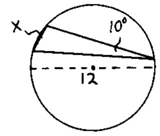
3) Fill in the *Table of Sines* below. Some of the answers can be found this set, or on the previous problem set. Others you will have to calculate.

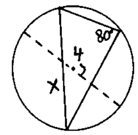
Table of Sines (and Cosines)

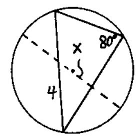
α	sin(α)
0°	
10°	
20°	
30°	
40°	
45°	
50°	
60°	
70°	
80°	
90°	
100°	
110°	
120°	
130°	
135°	
140°	
150°	
160°	
170°	
180°	

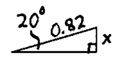
4) Use the *Table of Sines* to find x.

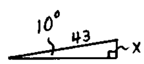
a) 

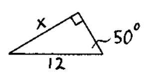
b) 

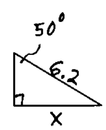
c) 

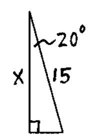
d) 

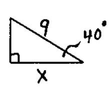
e) 

f) 

g) 

h) 

i) 

j) 

Algebra Review Test

Simplify. (2 points each)

1) $7x^4 + 2x^4$

2) $(6x^3)(7x^3)$

3) $(x - 9)^2$

4) $(2x^2)^3$

5) $3+4(x^2 + 7x - 5)$

6) $5x^3 + 4x^2 - 8x^3 + 7y$

7) $(2x^2 + 3)(5x^2 - 1)$

Simplify. (2 points each)

8) $\frac{9x^5z^{-2}}{15x^3y^{-3}z^8}$

9) $5x^3y^2(3x^4y + 5x^3z^3)$

Evaluate (2 points)
given that $x = -5$; $y = -3$.

10) $yx^2 - \frac{6}{y} + 3x$

Factor. (2 points each)

11) $x^2 - 17x + 60$

12) $x^2 + 17x - 60$

13) $x^4 - 16$

14) $x^2 - 25y^2$

15) $5x^6 + 25x^5 - 30x^4$

16) $12x^7y^5z^5 - 15x^2y^8$

Find the Common Solution. (4 points)

17) $3x - 4y = 19$
$5x + y = 1$

Solve. (4 points each)

18) $7 - 5(x + 4) = 2x - 4 + x + 31$

19) $3x^2 - 7x = 4x^2 + 3x + 21$

20) $\frac{5}{x + 3} = \frac{4}{2x - 7}$

21) $6x^2 + 4x = 2x + 10$

22) $-5x + 68 = (x - 4)(x - 7)$

23) *Challenge!* (2 points)

Simplify $\dfrac{1}{1 - \dfrac{x+3}{x - \frac{9}{x}}}$