10th Grade Assignment – Week #15

<u>Announcements</u>:

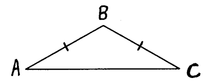
- Our work with proofs (probably the biggest theme of the year) is culminating in the next few weeks.
- I will give you a test on "Proofs" as part of next week's assignment. This test will be a collaborative effort - taken as a group. Your task on the test will be to write two (short) proofs.
- "Presentation Week" starts in two weeks! During Week #17 (January 18-22), students (mostly in pairs, or groups of three) will give their presentations. Your task during this week will be two-fold: (1) to give your presentation, and (2) to attend your classmates' presentations. (And, yes, I will also give a (recorded) presentation on the proof of Heron's formula.)
- During these next two weeks, if you are not doing a presentation, or if your group would like to have some extra material to work on, there are some (optional) "extra problems" for you to work on.
- For more details about the presentations, see "Guidelines for the Presentations", given on last week's assignment.

<u>Individual Work</u>

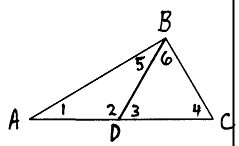
- Proof practice problem for the upcoming test, from **Problem Set #4** (*Proofs* unit),
  do these problems in this order: #11, 15, 6, 16.
- *Extra Problems (enough for the next three weeks, if you really have the time and desire.).*
  From **Problem Set #4**: problems #5, 10, 14, 8, 17, 18, 19, 9
  From **Problem Set #5**: <u>Part I</u> (Pentagon & Golden Triangle): do only #1-6
  <u>Part II</u> (All triangles are Isosceles)
  <u>Part III</u> (Pythagorean Curiosity): Answer any of the questions you wish.

Problem Set #4

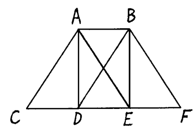
1) a) Find ∠A if ∠B = 118°.
   b) Find ∠A if ∠B = x°.
   c) Find ∠B if ∠A = y°.



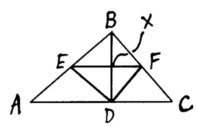
2) Given BD = AD = DC
   a) Find ∠ABC if ∠1 = 34°.
   b) Find ∠ABC if ∠1 = x°.



3) Given: Rectangle ABED; ▱ ABDC.
   Prove: ΔACE is isosceles.

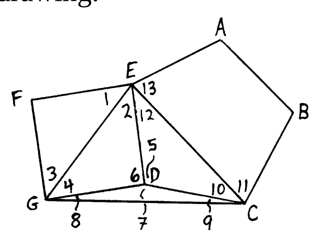


4) Given: E and F are midpoints of AB and BC, BD ⊥ AC, and that DF=15, CD=18, AD=32, AE=20.

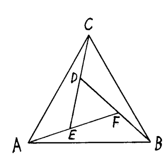


   a) Find the length of all the missing line segments.
   b) If ∠AED and ∠DFC are both bisected, thereby dividing those two triangles into two smaller triangles, then how many differently shaped non-divided triangles are there?
   c) If ∠BAC ≈ 36.9°, find all the angles in the original drawing.

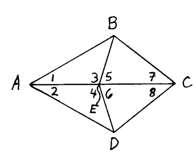
5) Given that ABCDE is a regular pentagon and that DEFG is a square, find all the angles in the drawing.



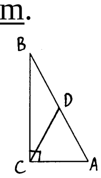
6) Given: ΔABC is equilateral; ∠ACE ≅ ∠BAF ≅ ∠CBD.
   Prove: ΔDEF is equilateral.



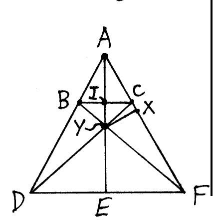
7) Write a paragraph proof to prove that if AC bisects both ∠BAD and ∠BCD, then it also bisects ∠BED.



8) **Midpoint of the Hypotenuse Theorem.
   "In a right triangle, a line drawn from the right angle to the midpoint of the hypotenuse creates two isosceles, equal-area triangles." Prove it.

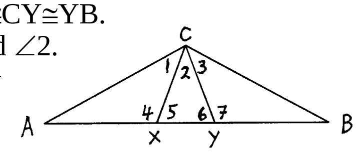


9) **Given: ΔADF is equilateral; DF ⊥ AE; BC ⊥ AE; XY ⊥ AF; DF = 1; BD = 2AB.
   Find XY and CY.

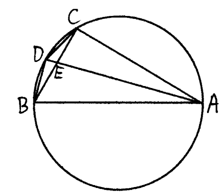


10) Given: ∠1≅∠2≅∠3; CX≅CY≅YB.
   Find: The measure of ∠6 and ∠2.

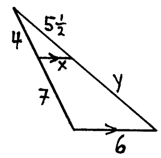
11) Given: ∠1 ≅ ∠3; CX ≅ CY
   Prove: ΔABC is isosceles.



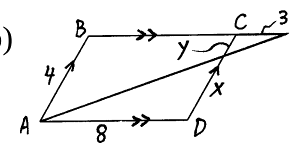
12) ***From Archimedes' derivation of π*
   Given: AB is a diameter of the circle; arcBD = arcDC.
   Prove: AD:BD = (AB+AC):BC.



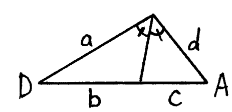
13) Find x and y.
   a)



   b)

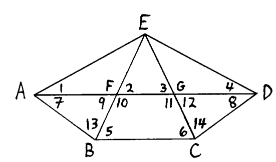


14) a) Given d=4, c=3, a=6, find b.
   b) Given d=4, c=3, b=5, find a.
   c) Given d=4, a=6, DA=8, find c.

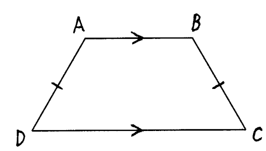


15) Given: ∠5 ≅ ∠6; ∠1 ≅ ∠4; AB ≅ CD.
   Prove: ΔABE ≅ ΔDCE.

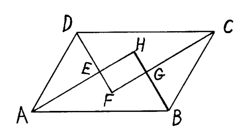
16) **Given: ∠2 ≅ ∠3; ∠1 ≅ ∠4; BC || AD.
   Prove: ∠7 ≅ ∠8.



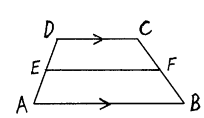
17) Write a paragraph proof to prove that with any isosceles trapezoid the base angles are equal.



18) **Parallelogram Angle-Bisector Th.
   "The angle-bisectors of any parallelogram form a rectangle." Prove it.

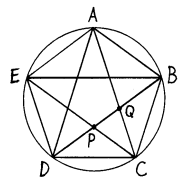


19) **Given: Trapezoid ABCD has median EF.
   Prove: EF || AB



Problem Set #5

I. ** The Pentagon & the Golden Triangle



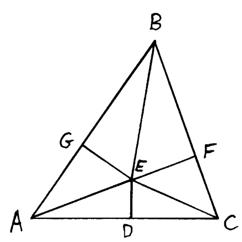
1) The drawing shows a pentagram inscribed inside a regular pentagon. How many differently shaped triangles are there?
2) Prove $\Delta DCQ \sim \Delta ACD$.
3) Find all the angles in the drawing.
4) Prove that $AQ^2 = AC \cdot QC$
5) Given CD = 1, find AQ, AC, QC, PQ (as decimals).
6) With any pentagon, what is the ratio (in decimal form) of the *diagonal to the side*? (For example, with a square it is approximately 1.414:1.) Also, what is the ratio of the *side to the diagonal*?

7) *The Golden Section.*

   a) Given that JL = 1, where must K be placed along JL such that *"the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment"*? (From *The Elements,* Theorem II-11)
   b) Find the ratio of the *whole to the larger segment* (in decimal form).
   c) Find the ratio of the *larger segment to the whole*.
   d) Find the ratio of the *larger segment to the smaller segment*.
   e) Find the ratio of the *whole to the smaller segment*.

II. ** All Triangles are Isosceles!?

This proof shows that all triangle are isosceles. Find the error!



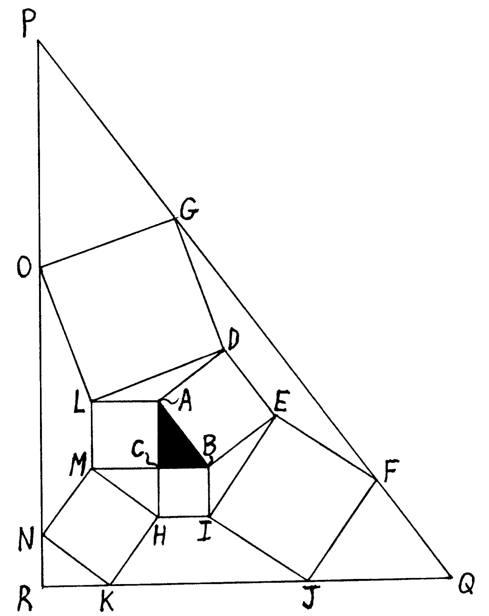
1. Let ΔABC be any random triangle.
2. Draw the perpendicular bisector of AC.
3. Draw the angle bisector of $\angle B$.
4. Let point E be the point of intersection of the perpendicular bisector of AC and the angle bisector of $\angle B$.
5. Draw EG such that it is perpendicular to AB. Draw EF such that it is perpendicular to BC.
6. $\Delta ADE \cong \Delta CDE$ because $AD \cong CD$, $\angle ADE \cong \angle CDE$, and DE is a shared side. Therefore $AE \cong CE$
7. $\Delta GBE \cong \Delta FBE$ because $\angle GBE \cong \angle FBE$, $\angle BGE \cong \angle BFE$, and BE is a shared side. Therefore $EG \cong EF$ and $BG \cong BF$.
8. $\Delta GAE \cong \Delta FCE$ because $EG \cong EF$, $AE \cong CE$, and $\angle BGE$ and $\angle BFE$ are both right angles. Therefore $AG \cong CF$.
9. $AB = AG + BG$ and $BC = CF + BF$.
10. $AB = BC$
11. **$\therefore$ ΔABC is isosceles!**

III. ** A Pythagorean Curiosity[1]

Construction:

Starting with the right triangle ABC, draw squares off each of the three sides. Connect the corners of those three squares (line segments LD, IE and MH) and then draw three more squares (NH, EJ and LG) off those line segments. Draw lines that connect the corners of these squares and extend the lines in order to form the triangle PQR.

Questions:



1) Which of the figures within the diagram (e.g. triangles, squares, etc.) are congruent? Which are similar?
2) Prove that PR is parallel to AC. (Likewise, it follows that CB is parallel to RQ.)
3) Find the area of each figure in terms of a and b, where a=CB and b=AC:
   a) ΔABC, ΔCHM, ΔADL and ΔIBE.
   b) The trapezoids HIJK, LMNO and DEFG.
   c) The squares drawn onto NO, FG, and KJ.
   d) The squares NH, EJ and LG.
   e) ΔJFQ and ΔOGP.
4) a) What is the relationship between the areas of the squares drawn onto ON, KJ, and FG?
   b) What is the relationship between the areas of the squares NH, EJ, and LG?
5) Comparing ΔABC and ΔPQR…
   a) Find a formula for the lengths of RQ and RP, in terms of a and b.
   b) Find the ratios of CB:RQ, AC:PR, and AB:PQ.
   c) Find the ratio of the area of ΔPQR to the area of ΔABC.

1. This construction is from page 252 of *The Pythagorean Proposition* by Elisha Loomis (NCTM, 1972). It is said to have first appeared in print in a New York newspaper in 1899. However, many of the questions are ours.