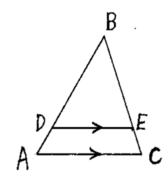
10^{th} Grade Assignment – Week #13

Notes:

- *Important!* In last week's assignment, as part of our foundation needed as we build up our own axiomatic system, I included these two important pages:
  (1) Summary of Topics Covered in the *Geometry Basics* unit
  (2) Definitions, Postulates, and Theorems (needed for the *Proofs* unit)
  It is important that you now add to this foundation by creating your own page in your own notebook, titled "**New Theorems**". This will consist of all the new theorems (which are important and useful enough to have name!) that we prove over the course of this unit. These new theorems can then be used to prove other new theorems in your assignments. For each of these new theorems, you should include a clear statement of the theorem. Start this page on your notebook now by adding these theorems:
  Rectangle Side Theorem, Parallelogram Angle Theorem, Parallelogram Side Theorem, Parallelogram Diagonal Theorem, Polygon Exterior Angle Theorem, Triangle Proportionality Theorem, etc., and any other theorem that we learn in the coming lessons.
- Some of the most important theorems from Euclid's *Elements, Book V* (some of which I went over in the lecture) are shown below.
- At the end of Monday's lecture, I went over (and proved) the Triangle Proportionality Theorem. Here is its statement:
  If a line is drawn parallel to one side of a triangle and intersects the other two sides, then it divides them into equal proportions. $BD : AD = BE : CE$



Group Assignment:

for Tuesday. Do the problems from **Problem Set #2** (*Proofs* unit) in this order:

- Do problems #1 and #4. It may help to use one of the theorems from Euclid's *Book V* (see below).
- Do the proof from problem #14.
- Use the Triangle Proportionality Theorem to do problems #6 and #7 (in the table).
- Do the proof from problem #16.

for Thursday

- Work on these proofs from **Problem Set #2** in the *Proofs* unit of the workbook:
  Do the proofs in this order: #11, 17, 18

Individual Work

- Do what you can with these problems from **Problem Set #2** (*Proofs* unit),
  in this order: #12, 2, 5, 8, 9, 10, 3
  …and if you really need an extra challenge, do #15 and #19
- Any proofs from the above Group Assignments that were not completed during your group meetings, can be worked on individually.

Euclid's *The Elements*, Book V (Laws of ratios and proportions)

Th. V-12: If a:a' = b:b' = c:c'.... then a:a' = (a+b):(a'+b') = (a+b+c...):(a'+b'+c'...)
Th. V-16: If a:b = c:d then a:c = b:d
Th. V-17: If a:b = c:d then (a−b):b = (c−d):d
Th. V-18: If a:b = c:d then (a+b):b = (c+d):d
Th. V-19: States that if a:b = c:d then a:b = (a−c):(b−d).

— Proofs —

Problem Set #2

Complete each statement. (From *The Elements*, Book V)

1) If $\frac{c}{3} = \frac{d}{2}$ then $\frac{d}{c} =$ ____

2) If $\frac{x}{a-7} = \frac{y}{3}$ then $\frac{x}{y} =$ ____

3) If $x{:}y = 2{:}5$ then $3x{:}4y =$ __

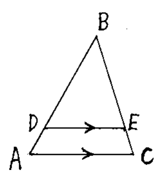
4) If $\frac{x}{a} = \frac{y}{3}$ then $\frac{x-a}{a} =$ ____

5) If $\frac{x}{5} = \frac{4}{y} = \frac{z}{w}$

then $\frac{x+4+z}{5+y+w} =$ ____

Use the Triangle Proportionality Theorem to fill in the table given the drawing shown here.

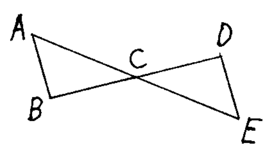
	BD	AD	AB	BE	CE	CB
6)	12	9		16		
7)		9	36			48
8)	4		5	6		
9)			24	20	10	



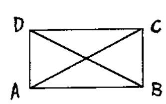
10) How can it be shown that the proportions $\frac{a+b}{b} = \frac{c+d}{d}$ and $\frac{a}{b} = \frac{c}{d}$ are equivalent?

11) <u>Given</u>: AB ≅ DE; AB ∥ DE.
<u>Prove</u>: ΔABC ≅ ΔEDC.

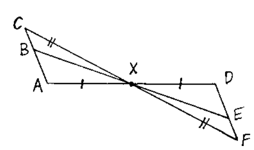
12) <u>Given</u>: C is the midpoint of AE;
C is the midpoint of BD.
<u>Prove</u>: ΔABC ≅ ΔEDC.



13) <u>Rectangle Diagonal Theorem</u>.
What property can be stated regarding the diagonals of a rectangle? Prove it.



14) <u>Given</u>: As indicated in the drawing.
<u>Prove</u>: BX ≅ XE.

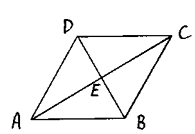


15) <u>Rhombus Diagonal Theorem</u>.

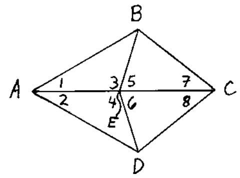
a) What property can be stated regarding the diagonals of any rhombus? Prove it.

b) Given rhombus ABCD, and AB = BD = 8, find the area of the rhombus, and the area of the rectangle formed by BD and AC?

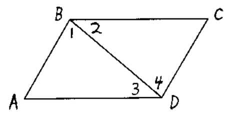
c) For any rhombus, what is the ratio of the area of the rhombus to the area of the rectangle formed by the diagonals?



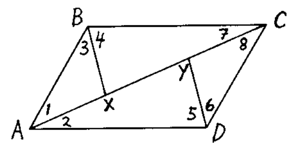
16) Given: AB ≅ AD; BC ≅ CD.
Prove: BE ≅ ED.



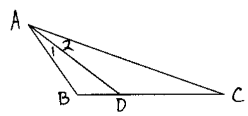
17) Given: AB || CD; AB ≅ CD.
Prove: BC || AD.



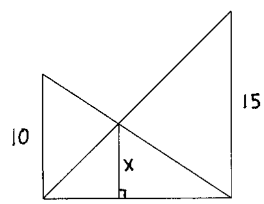
18) Given: ▱ABCD; ∠3 ≅ ∠6.
Prove: ΔADY ≅ ΔCBX.



19) **Triangle Angle–Bisector Theorem.
(from *The Elements*, Theorem VI-3)
"An angle-bisector of a triangle divides the opposite side into segments proportional to the other two sides." Prove it.



20) ** With the diagram shown here, the lines marked as 10, 15, and x are all parallel. It is interesting to note that if the base of the figure were lengthened or shortened, the value of x would not change. Find x.



21) **Eratosthenes' Measurement of the Earth

Around 230 B.C., Eratosthenes calculated the circumference of the earth by using only two measurements. First, at summer solstice, when the sun could shine to the bottom of a well in Syene (which is on the Tropic of Cancer), he measured that the sun at Alexandria (nearly directly north of Syene) was 7.2° off from being directly overhead. Secondly, he calculated, based upon the time it took a camel train to go from Alexandria to Syene, that the distance between the two cities was 5000 stadia.

a) What was Eratosthenes' value for the earth's circumference?
b) Given that the actual (average) radius of the earth is about 6371km, and that 1 stadium is believed to be about 157m (and this is disputed!), what was Eratosthenes percent error?