10th Grade Assignment – Week #12

Announcements:

- For this *Proofs* unit, it will be helpful to refer to the pages titled *"Summary of Topics Covered"* and "*Definitions, Postulates, and Theorems"*, both of which are found below.
- The *Answer Key* for this unit will likely be helpful to you, but keep in mind that there are many different ways to prove something.
- *Proof Presentations* will take place in Week #17.
  Here is everything that you need to know for now:
  - Each presentation will be done by a team or two or three students. Together, you will teach a class to your peers. This is a great educational opportunity for you!
  - The topics for the presentation will be assigned two weeks from now (week #14).
  - I will assist everyone giving a presentation to make sure that all questions are answered.
  - All presentations should be between 30 and 40 minutes, and will be done via a Zoom meeting.
  - More details will come once the date gets closer.

Group Assignment:

for Tuesday.

- Work on these proofs from **Problem Set #1** in the *Proofs* unit of the workbook:
  Do the proofs in this order:
  - **#9** *Parallelogram Angle Theorem*
  - **#13** *Rectangle Side Theorem*
  - **#6** *Polygon Exterior Angle Theorem* (This one is not a proof.)

for Thursday

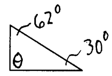
- Work on these proofs from **Problem Set #1** in the *Proofs* unit of the workbook:
  Do the proofs in this order:
  - **#10** (Not all proofs are worthy of a special name!)
  - **#11**
  - **#6** *Polygon Exterior Angle Theorem* (If you didn't already do it on Tuesday.)

Individual Work

- You should take the *Circle Geometry* test, found below.
- Any proofs from the above Group Assignments that were not completed during your group meetings, can be worked on individually.

Summary of Topics Covered in the Geometry Basics unit (In weeks #1-3)

- The angles in a triangle add to 180°.

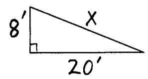


- The angles in a quadrilateral add to 360°.
- Simplifying square roots, such as:

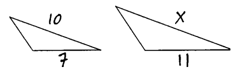
$$\frac{5}{\sqrt{3}} \rightarrow \frac{5\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} \rightarrow \boxed{\frac{5\sqrt{3}}{3}}$$

$$\sqrt{75} \rightarrow \sqrt{25\cdot 3} \rightarrow \sqrt{25}\cdot\sqrt{3} \rightarrow \boxed{5\sqrt{3}}$$

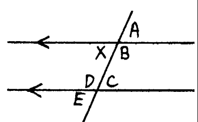
- Pythagorean Theorem



- Similar figure problems:



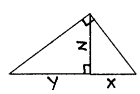
- Angle Theorems



  - Vertical angles are equal. (X, A)
  - Supplementary angles add to 180°. (X, B)
  - Corresponding angles are equal. (A, C)
  - **(Z Th.)** Alternate interior angles are equal. (X, C)
  - Same-side interior angles add to 180°. (X, D)

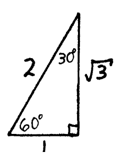
- Altitude of the Hypotenuse Theorem

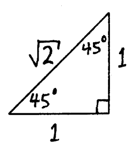
$$\mathbf{z^2 = x \cdot y}$$



- Special Triangles

The following two triangles appear frequently. The ratio of the lengths of their sides should be memorized.





- The ratio for any 45°-45°-90° triangle is $1:1:\sqrt{2}$
- The ratio for any 30°-60°-90° triangle is $1:\sqrt{3}:2$

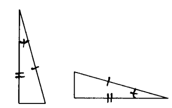
Example: Because the ratio for any 45°-45°-90° triangle is $1:1:\sqrt{2}$, this tells us two things:
(1) The hypotenuse is $\sqrt{2}$ times *longer* than the leg, and
(2) The leg is $\sqrt{2}$ times *shorter* than the hypotenuse.
It is critical to realize that once we know the ratio, it is not necessary to use the Pythagorean Theorem.
For example, if the leg of a 45°-45°-90° triangle is 5, then the hypotenuse is $5\sqrt{2}$. Also, if the hypotenuse of a 45°-45°-90° triangle is 7, then the leg is simply $\frac{7}{\sqrt{2}}$, which is also $\frac{7\sqrt{2}}{2}$.

Triangle Congruency Theorems

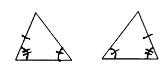
These congruency theorems answer the question: How can we know if two triangles are *definitely* congruent (equal)?
Example: If two given triangles each have sides of length 3, 6, 7, then these two triangles are definitely congruent. (This is the **SSS Δ ≅ Theorem**.)
Note how this would not be true with two quadrilaterals with sides of 5, 5, 8, 8. These could be two parallelograms with different angles, and therefore would not be congruent.

Here are the triangle congruency theorems:

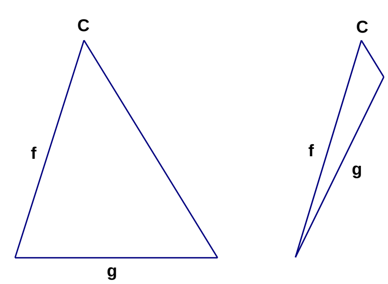
- **SSS Δ ≅ Th**. If all three sides of one triangle are congruent to the three sides of a second triangle, then the two triangles must be congruent.
- **SAS Δ ≅ Th**. If two sides and an angle between them in one triangle are congruent to two sides and an angle between them in another triangle, then the two triangles must be congruent.



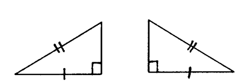
- **ASA Δ ≅ Th**. If two angles and a side between them in one triangle are congruent to two angles and a side between them in another triangle, then the two triangles must be congruent.
- **AAS Δ ≅ Th**. If two angles and a side not between them in one triangle are congruent to two angles and a side not between them in another triangle, then the two triangles must be congruent.



- **SSA is not a Theorem!!** To demonstrate why not, look at the two below triangles. They have two equal sides and an equal angle, *which is not between the equal sides*, but the triangles are clearly not congruent.



- **HL Δ ≅ Th**. If two right triangles have equal hypotenuses, and an equal leg, then the two triangles must be congruent. Note that this is a case of SSA that guarantees congruency.



Definitions, Postulates, and Theorems

needed for the *Proofs* unit

Any of the following may be used to justify a step of a proof:

Definitions

- Definitions from Euclid's *Elements*, *Book I*, including: *right angle, perpendicular, circle, equilateral, right triangle, isosceles triangle, square, rectangle, rhombus, parallel.*

Additionally, some definitions that Euclid didn't include in *The Elements*:

- If point K falls on line AB, and $\overline{AK} \cong \overline{KB}$, then we say that K is the ***midpoint*** of AB, and that K *bisects* AB.
- A ray that divides an angle into two congruent, adjacent angles is said to ***bisect*** that angle.
- With ***Congruent Figures***, all sides and angles are congruent.
- With ***Similar Figures***, all sides are in equal proportion and all angles are congruent.
- A ***Rectangle*** is a quadrilateral with four right angles.
- A ***Parallelogram*** is a quadrilateral with both pairs of opposite sides parallel.

Euclid's Common Notions

1. Transitive Property. Things which are equal to the same thing are also equal to each other.
2. Addition Property. If equals are added to equals, then the sums are equal.
3. Subtraction Property. If equals are subtracted from equals, then the differences are equal.
4. Things (i.e. figures or solids) that coincide with one another are congruent.
5. The whole is greater than the part.

Euclid's Postulates

1. A line can be drawn between any two given points.
2. Any line can be extended.
3. A circle can be drawn with any center and any distance [as its radius].
4. All right angles are equal.
5. The Parallel Postulate. If a line falling on two lines makes the interior angles on the same side less than two right angles, the two lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.

Postulates and Common Notions beyond what Euclid had:

- *Segment Addition Postulate*. Given point B falling on line segment AC: $\overline{AB} + \overline{BC} = \overline{AC}$.
- *Angle Addition Postulate*. Given point D falling inside ∠ABC: ∠ABD + ∠DBC = ∠ABC.
- *Substitution.* (Useful when working algebraically.) Example: If $x = 5a^2$ and $3x = 7$ then $3(5a^2) = 7$, etc.
- *Reflexive Property*. "Anything is equal to itself." This can be useful when proving congruent triangles.
- *Algebra*. When working with equations, our knowledge of algebra may be used to justify a step.

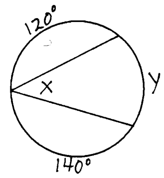
Theorems

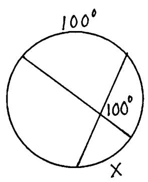
For the purpose of this unit, we will assume that the following theorems have been proven, most of which appear in Euclid's *Elements, Book I,* including:

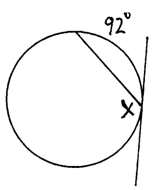
- *All Constructions* from Euclid's *Elements*, *Book I.*
- *Triangle Congruency Theorems* (SSS, SAS, ASA, AAS, HL)
- *Pythagorean Theorem* (and its converse)
- *Isosceles Triangle Theorem* (and its converse)
- *Triangle Interior Angle Theorem*.
  (Angles in a triangle add to 180°)
- *Triangle Exterior Angle Theorem*
  (The exterior angle of a triangle equals the sum of the two opposite interior angles.)
- *Angle Theorems* (Also, see above.)
  - Supplementary Angle Theorem
  - Vertical Angle Theorem
  - Corresponding Angle Theorem
  - Alternate Interior Angle Theorem
  - Same-Side Interior Angle Theorem
- **And, of course,** we also use any theorem that we ourselves have already proven.

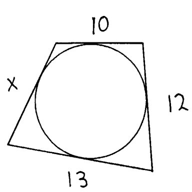
Circle Geometry Test

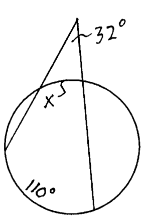
Each variable is worth 2 points, unless otherwise indicated.

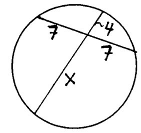
1) 

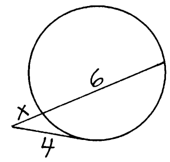
2) 

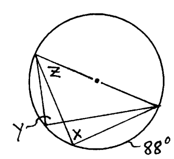
3) 

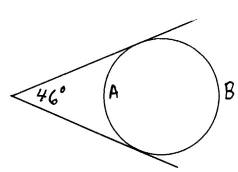
4) 

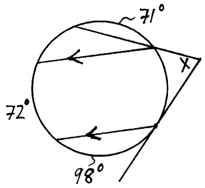
5) 

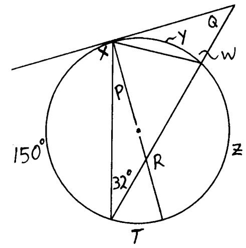
6)  4 points

7)  4 points

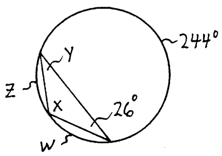
8) 

9) 

10) 

13) 

1 point each

11) 

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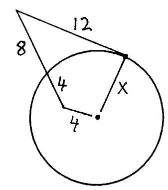
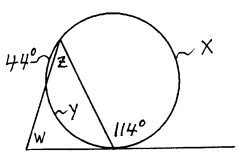
14) *Challenge!*
(Just for those who wish to try something extra difficult!)



12) 

1 point each

Problem Set #1

****Indicates a more challenging problem.**

Complete each statement. (From *The Elements*, Book V)

1) If $\frac{x}{5} = \frac{y}{4}$ then $\frac{x}{y} =$ ___

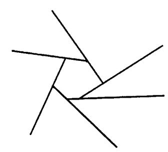
2) If $\frac{x}{3} = \frac{2}{y}$ then $xy =$ ___

3) If x:3 = 2:y then xy = ___

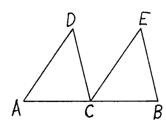
4) If $\frac{x}{2} = \frac{y}{3}$ then $\frac{x+2}{2} =$ ___

5) If $\frac{x}{y} = \frac{5}{3}$ then $\frac{x-5}{y-3} =$ ___

6) **Polygon Exterior Angle Theorem. We know that the sum of the interior angles in a triangle is 180°, that the sum of the interior angles in a quadrilateral is 360°, and that, in general, the sum of the angles in an N-gon is 180•(N-2). What can be said about the *exterior* angles of polygons? Make a statement for each kind of polygon, and then prove your statement(s).

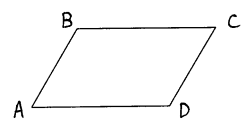


7) Given: C is the midpoint of AB; AD ≅ CE; AD || CE.
Prove: CD || BE.



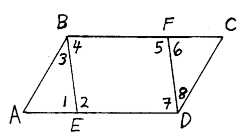
8) Parallelogram Side Theorem.
"Opposite sides of a parallelogram are congruent." Prove it.

9) Parallelogram Angle Theorem.
"Opposite angles of a parallelogram are congruent." Prove it.

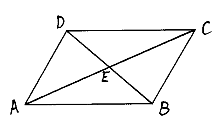


10) Given: ▱ABCD; AE ≅ FC.
Prove: BE ≅ FD.

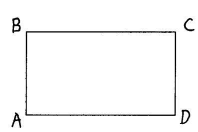
11) Given: ▱BEFD; AE ≅ FC.
Prove: ∠A ≅ ∠C.



12) Parallelogram Diagonal Theorem.
What property can be stated regarding the diagonals of a parallelogram? Prove it.



13) Rectangle Side Theorem.
"The opposite sides of a rectangle are congruent and parallel." Prove it.
(Note: Our definition of a rectangle does *not* state that it is a parallelogram.)



14) **Triangle Proportionality Theorem.
(from *The Elements*, Theorem VI-2)
"If a line is drawn parallel to one side of a triangle and intersects the other two sides, then it divides them into equal proportions." Prove it.