10^{th} Grade Assignment – Week #11

Announcements:

- I have decided to end our studies of *Circle Geometry* and *Triangle Geometry* after this week.
- For those who would like to do more with *Triangle Geometry*, I have included a few extra (amazing!) drawings in the "Individual Work", below. This is for those who have the extra time and desire.
- In the tutorial, everyone should bring their favorite drawing that they did from the Triangle Geometry unit, and be prepared to show it to their classmates.
- We will begin the *Proofs* unit (from the workbook) starting with Wednesday's lecture. You may wonder: how will this *Proofs* unit be different from the proofs we studied during the *Greek Geometry and Deductive Proofs* main lesson? In the main lesson, we studied proofs created by brilliant minds from thousands of years ago. You did not create your own proofs. In the upcoming *Proofs* unit, you will begin to learn the art of creating your own proofs. This is perhaps our greatest challenge this year. Often, my 10^{th} grade students (who put forth the required effort) feel that this is the highlight of the year. Be ready!
- Next week's assignment will include a test on *Circle Geometry*. (The test will not include *Triangle Geometry*.) You can best prepare for this test by making sure you understand all the problems on **Problem Sets #4-7** of the *Circle Geometry* unit.

Group Assignment:

for Tuesday.

- Work together (one problem at a time collaboratively!) on the problems from **Problem Sets #7** of the *Circle Geometry* unit. Save the last (challenge) problem for Thursday.

for Thursday

- Work together on problems #2 and #13 from **Problem Set #6** of the *Circle Geometry* unit.
- Help each other out with any of the other more difficult problems from **Problem Sets #5-7.**
- *Challenge!* Work together on problem #12 from **Problem Set #7**.

Individual Work

- From of the *Circle Geometry* unit, do **Problem Set #5** (all problems) and **Problem Set #6** (#3-12).
- *Triangle Geometry*. Do the following drawings:

1) ***The Nine-Point Circle.*** (Required!) (This was partly explained in Monday's lecture.)

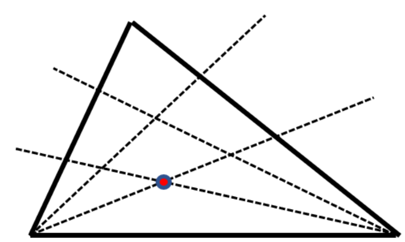
Note: It may be best to do two drawings, one with ΔRST (the original triangle) acute, and the other with ΔRST obtuse. Try to have ΔRST fairly large on the page, but be sure, if ΔRST is obtuse, that the circumcenter and orthocenter fall on the page.

Follow these instructions:

- Draw scalene (random) triangle RST.
- Find circumcenter C, and orthocenter O.
- Draw the line OC.
- Find the midpoints of RO, SO, and OT, and label them H, G, I, respectively.
- Label the feet of the altitudes on ΔRST as A, B, E.
- Label the midpoints of the sides of ΔRST as L, M, P.
- Label midpoint of OC as N.
- With N as the center, you can now draw a circle that passes through A, B, E, H, G, I, L, M and P!

2) ***Feuerbach's Theorem.*** (Required!) (This will be explained in Wednesday's lecture.)
This needs to be done super accurately. Expect it to take a few attempts before you get satisfactory results!
In ink, draw an acute scalene triangle (that takes up no more than ⅓ of the page) on a clean sheet of paper and extend the three sides so that they go to the edges of the paper. Construct the four in-circles and the nine-point circle. The circles should be in ink, if possible. Erase all construction lines. Now, repeat the same drawing, but start with an obtuse triangle.
Feuerbach's Theorem is a statement about the relationship of the five circles. What is that statement?

3) ***The Theorem of Morley.*** (Optional, for those who have extra time and desire)
Draw a large scalene triangle and, using a protractor, trisect each of the triangle's angles. (Recall that there is no method for trisecting an angle with a compas and straightedge.) With each of the six trisector lines, find its point of intersection with its closest trisector line from a neighboring point of the triangle. (The drawing at the right shows two trisected angles, and a red point of interesction between two neighboring trisector lines. I am not showing the third angle being trisected because I don't want to give it away!) In the end, you will have three new (red) points inside the triangle. What is special about these three points?

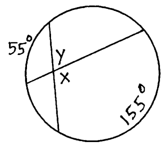


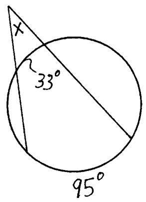
4) ***Simson's Theorem.*** (Optional, for those who have extra time and desire)
Draw three circles (as large as possible) on the same page. With the first circle, choose any three points on the perimeter of the circle and then connect these three points to form a triangle. Choose a fourth point, P, on the perimeter of the circle. From point P, construct a perpendicular line to each of the three sides of the triangle (you may have to extend the sides of the triangle), and label where each of these lines intersects the side of the triangle as points X, Y and Z. Repeat the same procedure for the other two circles, but use different triangles and a different point P. *Simson's theorem says something about these drawings.* What do you think it is?

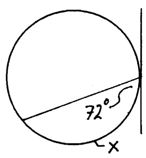
5) ***Steiner's Hypocycloid.*** (Optional – this one is amazing, but takes a lot of time!)
This drawing is the culmination of our study of triangles. It must be done carefully, and you must clearly understand what a Simson line is (from the previous problem) before doing this drawing.
Start with a circle (in ink) centered on the page that has a radius equal to about 25% of the width of your page. Draw any triangle (in ink) inscribed in that circle. In ink, construct the *Nine-Point Circle* and *Morley's Triangle* within your triangle. Erase everything but the two circles and the two triangles. Now for the demanding, but interesting, part. From a large number of points (can you do 60?) on the original circle, draw *Simson Lines* that extend to the borders of the page. Try to find short cuts for drawing the Simson lines, but maintain high accuracy. Make sure that only the Simson lines appear, not any construction lines, and, for best results, avoid drawing any lines through the Morley triangle. All of these Simson lines form a new shape, called a *hypocycloid*.
What is the relationship between the five forms (two triangles, two circles, and the hypocycloid) you have created?

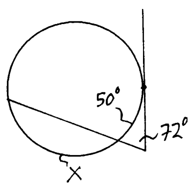
Problem Set #5

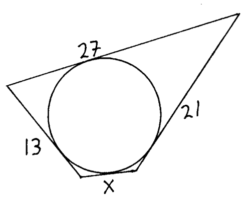
Circles

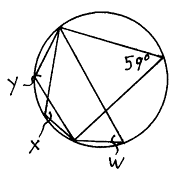
1) 

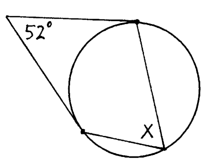
2) 

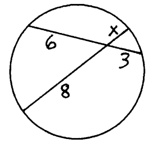
3) 

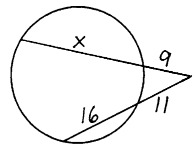
4) 

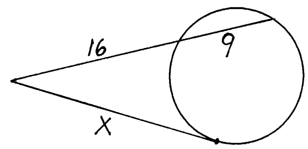
5) 

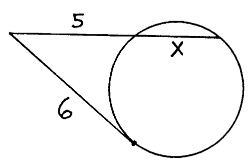
6) 

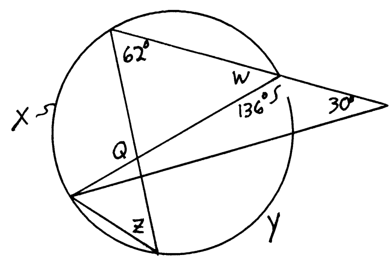
7) 

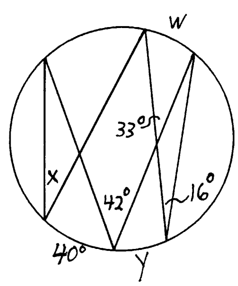
8) 

9) 

10) 

11) 

12) 

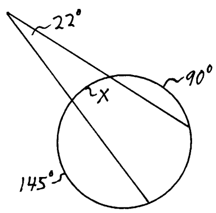
13) 

Problem Set #6

Group Work

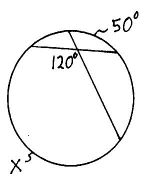
1) Draw a circle with a radius of about 3cm. Draw a secant segment (which is a chord that has been extended in one direction past the edge of the circle) so that both the internal and external portions each have a length of 4cm. Draw a line from the end of the external portion so that it is tangent to the circle. (Does it matter which side you draw the line?) Now calculate the length of that tangent segment, and measure it to see if your calculation was accurate. Repeat the whole process, but this time, use a circle with a radius of 5cm.

2) Find X.



Homework

3)



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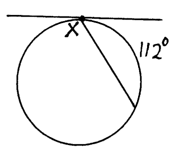

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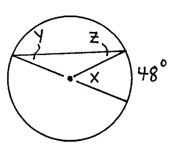

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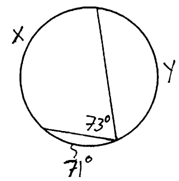

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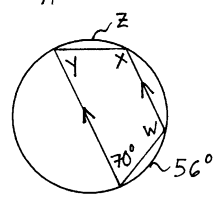

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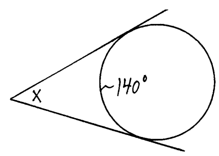

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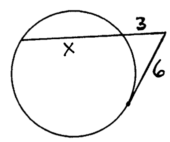

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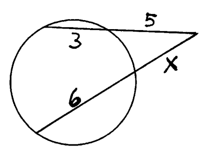

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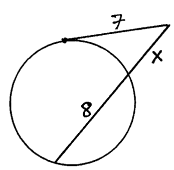

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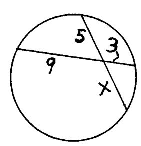

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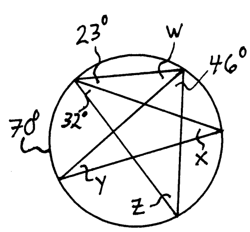
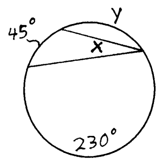
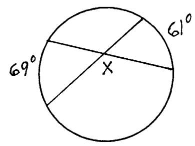
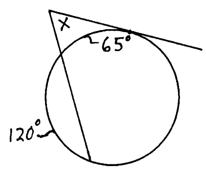


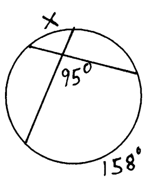
Problem Set #7

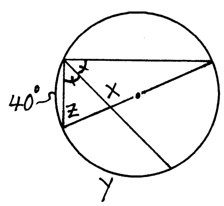
Circles

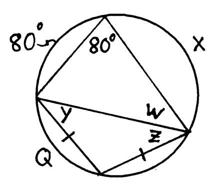
1) 

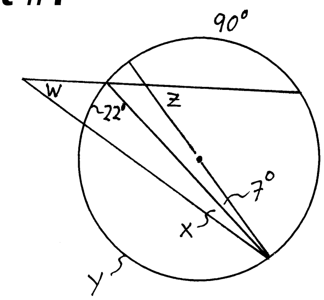
2) 

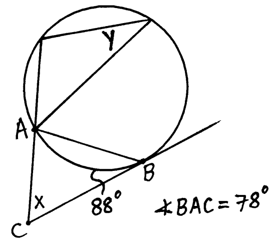
3) 

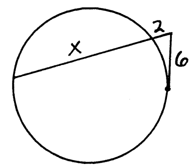
4) 

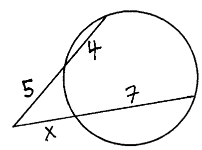
5) 

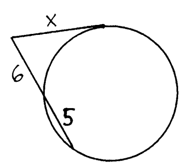
6) 

7) 

8) 

9) 

10) 

11) 

12) *Challenge!*
Find x given that arc AB equals arc AC.