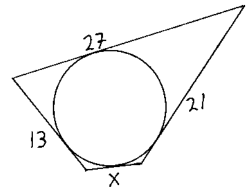
10^{th} Grade Assignment – Week #10

Group Assignment:

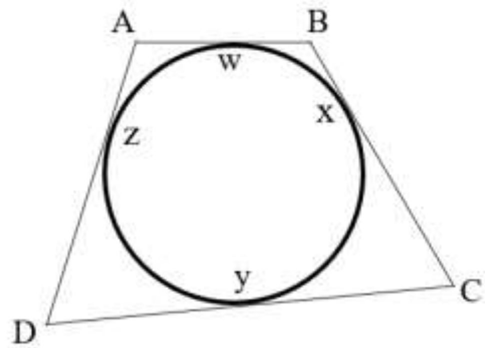
for Tuesday.

- **Circumscribed Quadrilateral Theorem**



Our goals here are:
- To discover the *Circumscribed Quadrilateral Theorem*, and
- To solve the "sample problem" shown on the right.

In order to discover the *Circumscribed Quadrilateral Theorem*, follow these steps:

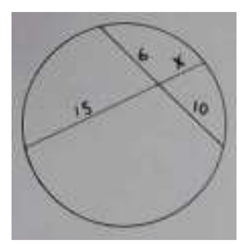


1. Given the quadrilateral ABCD (as shown on the right), which has all of its sides tangent to the circle, with points of tangency labeled as w, x, y, z.
2. Consider that each of the four sides of the quadrilateral are broken into two segments at the point of tangency. (Therefore, we now have eight segments.)
3. Together, discuss this question: Of these eight segments, which ones are exactly the same length? (Hint: Think of the *Intersecting Tangent Theorem*.)
4. Using colored pens or pencils, color the segments AW and AZ with the same color. Use a second color for the segments BW and BX, a third color for CX and CY, and a fourth color for DY and DZ. What do you notice? This now shows the answer to the previous step: segments of the same color have the exact same length!
5. You can now (hopefully!) discover the statement for the *Circumscribed Quadrilateral Theorem* by answering this question: **What can be said about opposite sides of the quadrilateral ABCD?**
6. Now use what you have (hopefully) discovered in order to solve the "sample problem", shown far above.

- **Secant Segment Theorem**

Our goals here are:
- To discover the *Secant Segment Theorem*, and
- To solve "Example #2" (shown below, on the right).

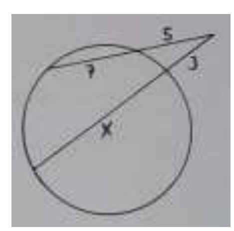
Example #1



Follow these steps:

1. In last week's Wednesday lecture (Week #9, Lecture #2), I introduced the *Chord Segment Theorem*. Remind each other what this theorem states.
2. Solve "Example #1" (shown above, on the right).
3. Remind each other how I proved the *Chord Segment Theorem* in the lecture.
4. Using a method similar to how I proved the *Chord Segment Theorem*, solve "Example #2". (Hint: Create two triangles by drawing two chords that connect the ends of the subtended arcs.)
5. The *Secant Segment Theorem* applies to secants that intersect outside the circle, such as with Example #2. **Give a statement for the *Secant Segment Theorem*.**

Example #2



Group Assignment:

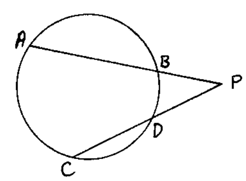
for Thursday

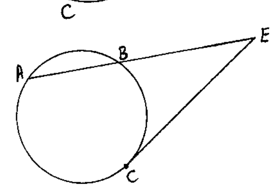
- **The Four Centers of a Triangle**

  Follow these steps:
  1. Study the next page "The Two Centers of a Triangle in Movement". For each triangle in the sequence, everyone should add points for the circumcenters (in blue) and orthocenters (in pink). You don't need to do this exactly with a compass and straightedge; you can just do it "by eye" as best you can. Note that two of these triangles are right triangles. Especially discuss where the two points are during the strange moments when the triangle is an "H" and when it degenerates into a single line.
  2. As I explained in the first lecture this week, you should try to very exactly imagine that this is a repeating, looping video (gif), where there are not just 8 frames, but millions of frames, in one cycle of the video. The sides of the triangles are rotating at constant and equal speeds, but in opposite directions. If you can really see the video accurately in your head (which is very challenging!), then you can answer the following questions:
  3. Assuming that the video is repeating over and over again…
     When does the orthocenter move slowly, and when does it move very fast?
     When does the circumcenter move slowly, and when does it move very fast?
     During one full cycle of the video, how many times do the two points pass each other?

- **Secant Tangent Theorem**

  Follow these steps:
  1. Make sure everyone understands that (with the drawing shown on the right) the *Secant Segment Theorem* can be summarized by this formula: AP•PB = CP•PD
  2. Now imagine that the line CDP rotates about P thereby causing points C and D to slowly come together and resulting in the second drawing. Line CE is now tangent to the circle, and this leads us to the *Secant Tangent Theorem*.
  3. Using the second drawing, state the *Secant Tangent Theorem* as a formula.





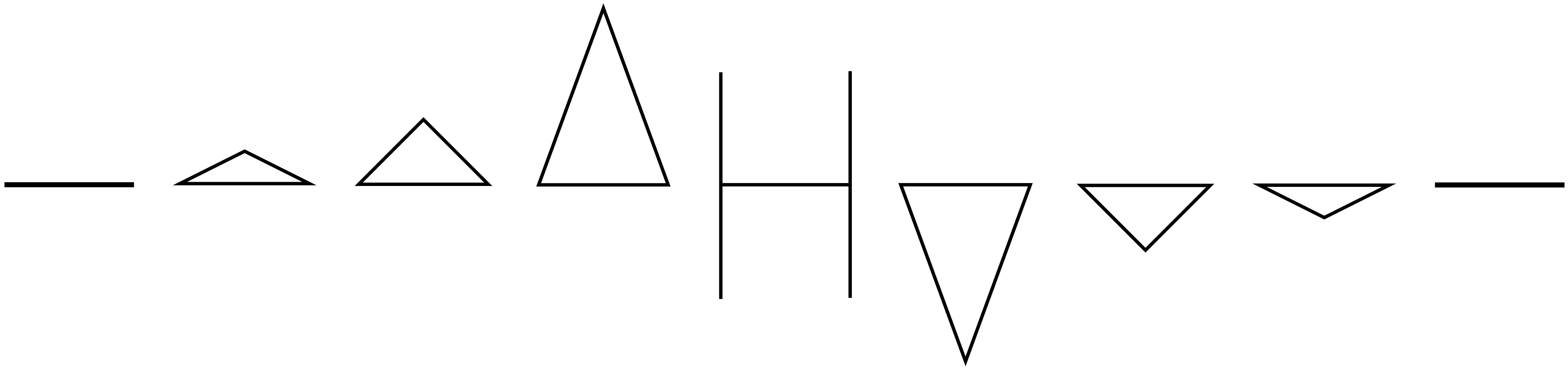
- **Practice** (If you have extra time.)

  Work together on some of the more difficult problems from **Problem Sets #3 and #4** of the *Circle Geometry* unit.

Individual Work

- Do **Problem Sets #3 and #4** of the *Circle Geometry* unit.
- **Finish up.** Continue working on the group assignments (from Tuesday and Thursday) that your group didn't complete.

The Two Centers of a Triangle in Movement



Problem Set #3

Find each variable.

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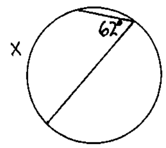

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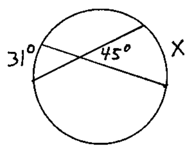

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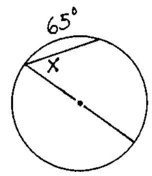

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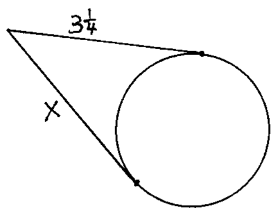

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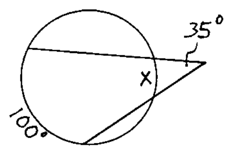

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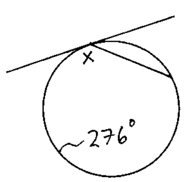

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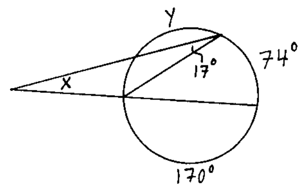

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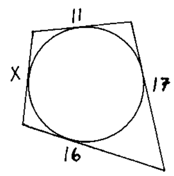

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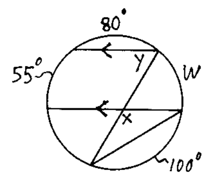

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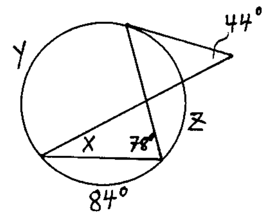

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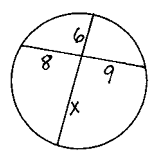
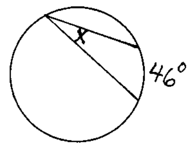


Problem Set #4

Circles

1)



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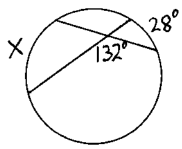

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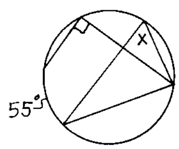

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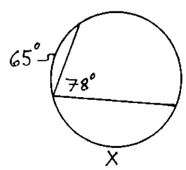

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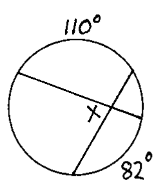

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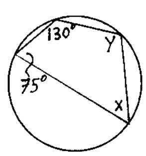

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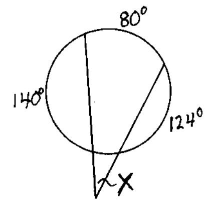

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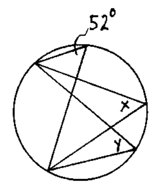

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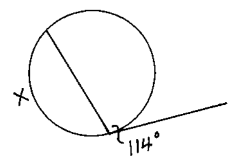

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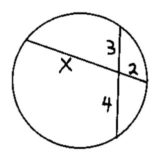

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