Tutorial Session Notes
Grade 9
Quarter #1 (Week 1-8)

<u>About these notes</u>:

- These notes are primarily for those who are acting as the tutor — either a parent or a class teacher.
- During the second year of JYMA, Mr. Messner (our JYMA tutor) kept scribbled records of some of the proceedings of his Friday tutorial sessions. These notes are a reconstruction of those scribbles.
- These lessons were often a spontaneous collaboration with the students present and incorporate their questions. In the process of clarifying details, examples occasionally stepped beyond the skills presented in lectures. This is not an ideal script, but only an offering of possible tutorial activities.
- In order to support those who are acting as the tutor for their child or a class, I am sharing these notes with those who are acting as the tutor.
- Of course, these tutorial sessions are also an opportunity for the students to ask their tutor questions.
- If you are acting as the tutor, it may be helpful to read the section of the JYMA handbook titled “The Role of the Tutor”.

Week #1-3

<u>Note</u>: In the Making Math Meaningful curriculum, algebra is first introduced during a 7th-grade main lesson, then after a significant sleep it reappears in a skills class unit at the end of 8th-grade. For those who had this background and took to it easily, the initial weeks of 9th-grade could be a breezy review. For those with less comfort or exposure, the amount of content and the speed at which it is covered is likely to feel intense. It is important to recognize that there will be opportunity to practice these skills all year (and beyond), that the pace with truly new material will be slower, and that during weeks 6-9, while lectures focus on the Descriptive Geometry main lesson, there will be additional time for students to focus on consolidating the skills presented during the initial weeks of the term. • During these initial weeks, tutorials should survey core pre-algebra and basic algebra skills, moving quickly where students are comfortable, and repeatedly revisiting areas where further growth is needed. A sampling of topics to review during these weeks (and later as need be) follow here. Adapt to your student(s).

- Contrast what Mr. York refers to as the middle school (or 8th-grade) method for solving equations versus the high school (or 9th-grade) method. Do not, however, move exclusively to the latter method too soon. Until the techniques are fully internalized, writing them out helps students learn.

- Solve an equation of the form “Ax + B = Cx + D” (choosing constants and coefficients as suits) using both of the above methods. Emphasize the need to understand how they are the same rather than being dogmatic about how students document their process. What is most essential is that they do document their process, writing out steps, rather than trying to do everything in their heads. (It is excellent mental work to try to do everything in one’s head, but our goal here is to develop such good written process as to be able to successfully undertake problems beyond what anyone could hold in their head, and to be able to read and understand every step of the process after the fact.)

- Review the Order of Operations. Give particular attention to the following:
  - The P for Parentheses of "PEMDAS" would be better designated G for Groups.
  - Grouping symbols include parentheses, brackets, absolute value signs, radicals, and fraction bars.
  - Something of the form… $\frac{A+B}{C+D}$ requires completing the addition in the numerator and denominator before dividing their sums. This seems to defy the order of operations until one recognizes that the fraction bar is a grouping symbol, and that the fraction as a whole could thus be written on one line as… $(A+B)\div(C+D)$.
  - Parentheses are not operations of themselves, but absolute value signs or fraction bars are simultaneously grouping symbols and operations.
  - The Order of Operations is best thought of as occurring in levels.
    (0) Groups
    (1) Exponents
    (2) Multiplication / Division / Negation
    (3) Addition / Subtraction
  - Operations within the same level are given equal precedence, and therefore are executed from left to right. Provide an example of the form "A ÷ B × C" and note that the division is done first.
  - The inclusion of Negation in the order helps to show why…
    $(-A)^B \neq -A^B$ -and- $-A^B = -(A^B)$
  - The Order of Operations is not arbitrary. Exponents precede multiplication because exponents are a shorthand means of expressing repeated multiplication. Multiplication likewise precedes addition because multiplication is a shorthand means of expressing repeated addition.

- Review the many ways multiplication can be designated …
  $$A \times B \times C = A \cdot B \cdot C = A(B)C = (A)(B)(C) = A(B(C)) = ABC$$

- Review the concept of algebraic terms. Key understandings here are that "terms are separated by addition", "subtraction means adding the opposite", "the negative is part of the number", "the negative is part of the term", "multiplication binds parts of a term together", and the terminology "variable", "coefficient", "constant", "exponent", and "radical".

- Review "like terms". When combining terms, "only change the number (coefficient), never change the name (variable part)".

- Note that the checkbook example Mr. York discussed is a good way to keep ideas about negatives concrete. Subtraction is not commutative, but as long as we keep the negative attached to the term, move around credits and debits, we are within the realm of correct commutative addition. (It can be helpful in algebra to replace the idea of adding and subtracting with the concept of combining. Rather than adding and subtracting numbers, we instead combine positive and negative terms.)

- Explain that the proper way to represent a negative fraction is to have the negative to the side of the fraction bar. If a negative appears in the numerator or denominator, technically we do not have a rational number, but instead an expression that is not fully simplified.

- Determine which of the following are equivalent, and which are proper fractions …
  $\frac{2}{7}$, $-\frac{2}{7}$, $\frac{-2}{7}$, $\frac{2}{-7}$, $\frac{-2}{-7}$, $-\frac{-2}{-7}$, $-\frac{-2}{7}$, $-\frac{2}{-7}$

- Consider (−1) raised to various powers. Arrive at the understanding that …
  $(-)^{even} = (+)$ -and- $(-)^{odd} = (-)$

- Note that any group can be considered a variable. For example, “3(2x+7)” can be thought of as “3G”. Thus we can consider “3(2x+7)” to function as a top-level term. Presenting this idea too early on can create confusion. But offered in context at the right time can be very helpful for some.

- Encourage students to love fractions and strive to increase their comfort with them and accuracy in calculations. Note that many students continue to work on this throughout high school, though the sooner one achieves mastery the easier everything else becomes. It is not uncommon for seniors to do every calculus step of a problem correctly, only to reach a wrong answer due to fraction troubles. While reviewing fraction operations may be welcomed by some students, it is best not to keep most equation work early on free of them, so they can concentrate on the algebra proper.

- Review the basics of Simplifying Expressions. Here is a recipe:
  - Go to the innermost group.
  - Combine any like terms.
  - Distribute and dissolve the group.
  - Repeat until done.

- Review the basics of Solving Equations (one variable and without exponents). Here is a recipe:
  - Simplify each side of the equation separately. (See above recipe.)
  - Move all variable terms to one side and combine.
  - Move all constants to the other side and combine.
  - Divide through by the coefficient of the variable term.

- During the initial weeks, continually remind students about how they can test their solutions. When time allows, go through this process with them by writing out each step. (Always replace variables with values in parentheses.) When time is short, if reasonable, make it a mental math exercise.

- Practice…
  - One-step equations (of form “Ax = B” and “x + A = B”)
  - Two-step equations (of form “Ax + B = C”)
  - Multi-step equations (of form “Ax + B = Cx + D”)
  - Multi-step equations (of form “Ax + B + Cx = D + Ex + F”)
  - Multi-step equations involving distributing (of form “A(Bx + C) …”)
  - Simplifying expressions or solving equations where a negative is distributed

- Review how Moving Along Diagonals can be used to solve equations of the form “A/B = C/D” where any of those variables could represent groups such as “2x+3”. But avoid quadratics!

- Review equations with, as Mr. York calls it, unusual solutions. That is, where x = 0, or all real numbers are solutions ($\mathfrak{R}$), or there are no real number solutions (∅). Note that zero is a valid number just like any other, so the first case might be unusual, but is not abnormal. The second two cases are more peculiar. When all variables disappear during solving, it implies that the solution does not depend on the value of the variable. This means then that either any value works, or that no value does. If we arrive at a true statement (such as 7 = 7), then all real numbers are valid solutions; seven will always equal seven, no matter what x is. If we arrive at a false statement (such as 5 = 8), then there is no solution; five will never equal eight, no matter what value for x we may choose.

- Review Exponent Rules (but not negative exponents yet) …

$\left(x^A\right)^B \rightarrow x^{AB}$	$\left(x^2\right)^3 \rightarrow x^6$
$x^A \cdot x^B \rightarrow x^{A+B}$	$x^2 \cdot x^3 \rightarrow x^5$
$\_x^A + \_x^A \rightarrow \_x^A$	$4x^2 + 5x^2 \rightarrow 9x^2$
$\_x^A + \_x^B \rightarrow \textit{cannot combine}$	$4x^2 + 5x^3 \rightarrow \textit{cannot combine}$

- Some students might appreciate calling the above Cloud Rules. Exponents can be thought of as appearing smaller than other numbers because they are far away, up in the clouds. Because they are so high up, when they interact they follow one rule lower down in the order of operations than the numbers on the ground do. So when a term is raised to an exponent, the exponents multiply. When terms are multiplied, the exponents are added. And when terms are added, the exponents do not change; there is no operation below addition.

- Create and practice basic examples of the above.

Week #4

Note: Anything from the above Week #1-3 notes not yet covered can be introduced here or thereafter, and anything that has been introduced can be reinforced as needed.

- Simplify: $-5(2x) + 2(-3y) - 4 \rightarrow -10x - 6y - 4$

- Solve: $-\frac{3}{5}x = -90 \rightarrow x = 150$

- Create equations in two or more variables and then solve for one in terms of the others. (For example, start with something of the form "Ax + By = C" and then solve for x in terms of y, as well as for y in terms of x. Then vary the form, and/or add z-terms.)

- Explain that / show why …

$$\frac{2x}{5} \Leftrightarrow \frac{2}{5}x \qquad \text{-and-} \qquad \frac{2x+5y}{9} \Leftrightarrow \frac{2}{9}x + \frac{5}{9}y \Leftrightarrow \frac{1}{9}(2x+5y)$$

- Recall the essential rule of evaluating expressions: replacing variables with values in parentheses while leaving everything else otherwise intact.

- Given $a = -7$, $b = 3$, and $c = -2$, evaluate: $a - 2bc$ → 5
- Given $f = -6$, evaluate: $\frac{2}{3}f - \frac{2}{5}(f - 4)$ → 0
- Solve: $7x - (5x + 4) = 2[8 - 2(9 - x)]$ → $x = 8$
- Create basic exponent problems for further practice.

Week #5

- Simplify: $(3x^3y)^2$ → $9x^6y^2$
- Simplify: $(3x^3 + y)^2$ → $9x^6 + 6x^3y + y^2$
- Exponents and Polynomials Problem Set #5 Q38 …
  Simplify: $(7x^5y^2 - 10xy^4)^2$ → $49x^{10}y^4 - 140x^6y^6 + 100x^2y^8$
- Exponents and Polynomials Problem Set #5 Q32 …
  Given $x = \frac{2}{3}$, and $y = -\frac{1}{2}$, evaluate: $xy^2 - \frac{1}{x} - \frac{x^2}{2y^3}$ → $\frac{4}{9}$
- Exponents and Polynomials Problem Set #4 Q40 …
  Solve: $\frac{x+1}{x+2} = \frac{x+3}{x+4}$ → $\varnothing$

Week #6

- Solve an equation such as "5x − 8y = 3" for y, then explain / show that…
  $\frac{-5x+3}{-8}$ …is equivalent to the preferred form… $\frac{5x-3}{8}$
  …which is also equivalent to… $\frac{5x}{8} - \frac{3}{8}$ …as well as… $\frac{5}{8}x - \frac{3}{8}$.
- Exponent practice. Create expressions such as AB^3 and $A(B-C)^2$ with monomials A, B, and C.
- Equation practice. Create an equation of form $\frac{Ax+B}{Cx+D} = E$ with constants A, B, C, D, and E.

- Descriptive Geometry? Draw a cross to create space for the three principal views. Place a yellow and blue dot (or small filled-in and outlined circle) in each view. Ask students to identify their relative positions (to the left of or the right, above or below, in front of or behind). For more advanced practice, provide only two of the principal views and ask the same questions.

Week #7

- Solve: $\frac{4x}{x-4} + 5 = \frac{5x}{x-4}$ → $x = 5$
  Ensure students understand how it can both be solved by (a) multiplying all top-level terms in the equation (of which there are three) by x−4, or (b) combining the common-denominator fractions and then cross-multiplying or moving along diagonals.

- Exponents & Polynomials Problem Set #7 Q28 …
  Solve: $\frac{1}{2}(x+2) = \frac{2}{3}(3x+9)$ → $x = -\frac{10}{3}$
  Ensure students understand how it can be solved by (a) distributing the fractions and proceeding per the usual recipe, or (b) clearing fractions by multiplying both sides of the equation by 6, the LCD. Caution students that we must only multiply the top-level terms. (Recall that the equation can be thought of as ½G = ⅔H.) If that all seems fuzzy to anyone, they should stick with the usual method.

- Exponents & Polynomials Problem Set #7 Q35 …
  Simplify: $\frac{6x^3y^8}{15x^6y^2}$ → $\frac{2y^6}{5x^3}$

- Exponents & Polynomials Problem Set #7 Q36 …
  Simplify: $\frac{(-2x^3y^2)^3}{(-4xy^3)^2}$ → $-\frac{x^7}{2}$

- Explain basics of simplifying square roots of squared monomials. (E.g. $\sqrt{100x^{10}}$ → $10x^5$)

[This space intentionally left blank.]

Week #8

- Review Exponent Rules, expanding to include division, and developing negative exponent rules.

$(x^A)^B \rightarrow x^{AB}$ $\quad\quad$ $(x^2)^3 \rightarrow x^6$

$\left(\frac{x^A}{y^B}\right)^C \rightarrow \frac{x^{AC}}{y^{BC}}$ $\quad\quad$ $\left(\frac{x^3}{y^4}\right)^2 \rightarrow \frac{x^6}{y^8}$

———————————————————————————————————

$x^A \cdot x^B \rightarrow x^{A+B}$ $\quad\quad$ $x^2 \cdot x^3 \rightarrow x^5$

$\frac{x^A}{x^B} \rightarrow x^{A-B}$ $\quad\quad$ $\frac{x^7}{x^4} \rightarrow x^3$

———————————————————————————————————

$x^{-A} \rightarrow \frac{1}{x^A}$ $\quad\quad$ $x^{-8} \rightarrow \frac{1}{x^8}$

$\frac{1}{x^{-A}} \rightarrow x^A$ $\quad\quad$ $\frac{1}{x^{-8}} \rightarrow x^8$

$\left(\frac{x^A}{y^B}\right)^{-1} \rightarrow \frac{y^B}{x^A}$ $\quad\quad$ $\left(\frac{x^2}{y^9}\right)^{-1} \rightarrow \frac{y^9}{x^2}$

$\left(\frac{x^A}{y^B}\right)^{-C} \rightarrow \left(\frac{y^B}{x^A}\right)^C$ $\quad\quad$ $\left(\frac{x^2}{y^9}\right)^{-4} \rightarrow \left(\frac{y^9}{x^2}\right)^4$

———————————————————————————————————

$x^0 \rightarrow 1$ $\quad\quad$ $6^0 \rightarrow 1$

———————————————————————————————————

$\_x^A + \_x^A \rightarrow \_x^A$ $\quad\quad$ $4x^2 + 5x^2 \rightarrow 9x^2$

$\_x^A + \_x^B \rightarrow$ *cannot combine* $\quad\quad$ $4x^2 + 5x^3 \rightarrow$ *cannot combine*

$\_x^A + \_y^A \rightarrow$ *cannot combine* $\quad\quad$ $4x^2 + 5y^2 \rightarrow$ *cannot combine*

- Practice problems involving negative exponents, as well as monomial over monomial division.

- Review simplifying square roots, now including cases where the monomial under the square root is not a perfect square (so that either a constant, variable, or both must remain in the house). Reinforce the rule that a simplified radical will never have an exponent in the house.