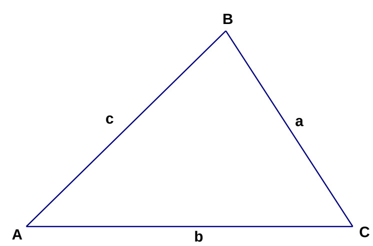
Geometry Basics ANSWERS

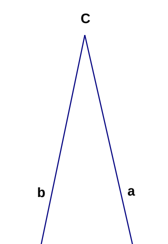
NOTE: All triangles are fairly precise but are approximations.

Problem Set #1

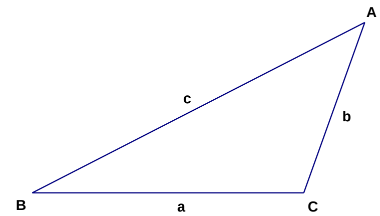
1) Supplementary.
2) Alternate Interior.
3) Vertical.
4) Same-side Interior.
5) Corresponding.
6) Congruent.
7) Add to 180°
8) Congruent
9) Congruent
10) Add to 180°
11) Add to 180°
12) A ≈ 44.4°, B ≈ 78.5°, C ≈ 57.1°



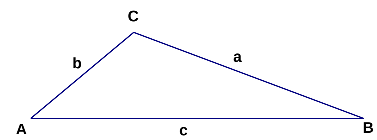
13) A ≈ 77.6°, B ≈ 77.6°, C ≈ 24.8°



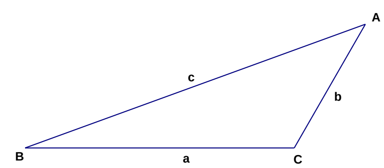
14) c ≈ 8.27 cm, A ≈ 43°, B ≈ 27°



15) a ≈ 5.2 cm, b ≈ 2.77 cm, C = 120°



16) a ≈ 5.2 cm, b ≈ 2.77 cm, B = 20° (Same as 15)
17) b ≈ 3.73 cm, c ≈ 9.43 cm, C = 120°



18) $x = 28°$
19) $x = 130°$
20) $x = 95°$
21) $x = 15°$, $y = 165°$, $z = 15°$
22) $f = 102°$, $h = 78°$, $j = 78°$, $k = 102°$, $m = 102°$, $n = 78°$, $p = 78°$
23) $x = 23°$
24) $\theta = 60°$
25) $x = 105°$, $y = 75°$, $z = 105°$
26) $2\sqrt{2}$
27) $\frac{4\sqrt{7}}{7}$
28) $10\sqrt{3}$
29) $\frac{\sqrt{6}}{2}$

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30) $2\sqrt{7}$

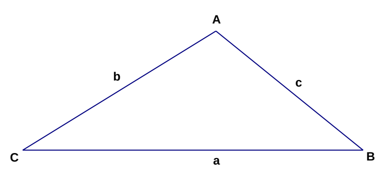
31) $\frac{\sqrt{10}}{5}$

32) $3\sqrt{11}$

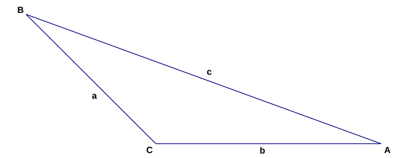
33) $\sqrt{5}$

Problem Set #2

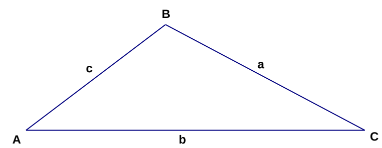
1) The sum of the angles of a quadrilateral is 360°.
2) The sum of the squares of the two legs of a right triangle is equal to the square of the hypotenuse.
3) $x = 10$
4) $x = 48$ ft.
5) $x = 4\sqrt{29} \approx 21.54$ ft.
6) 60°
7) Examples: A & E; D & H
8) Examples: A & D; F & G
9) Examples: C & D; E & G
10) D & F; C & E
11) D & E; C & F
12) $x = 110°$
13) $x = 97°$
14) $x = 132°$, $y = 48°$, $z = 48°$
15) $x = 50°$
16) $x = 95°$
17) $\theta = 57°$
18) $\theta = 88°$
19) $x = 70°$, $y = 70°$
20) $x = 70°$, $y = 110°$
21) $x = 66°$, $y = 66°$, $z = 114°$
22) $x = 85°$
23) A ≈ 109.5°, B ≈ 39°, C ≈ 31.5°



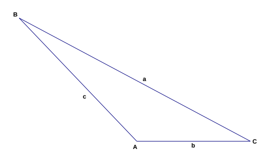
24) Impossible triangle. The sum of two of the sides is less than the third side.
25) a ≈ 6.47 cm, c ≈ 13.4 cm, B = 25°



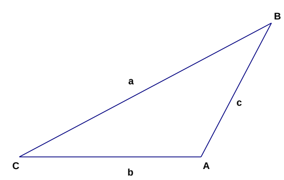
26) c ≈ 4.65 cm, A ≈ 37.3°, B ≈ 114.7°



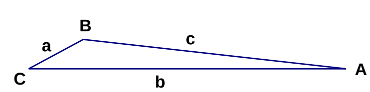
27) a ≈ 13.84 cm, A ≈ 133.76°, B ≈ 18.24°



28) Two possible triangles:
    a ≈ 9.43 cm, A ≈ 117.7°, B ≈ 34.3°



    a ≈ 1.17 cm, A ≈ 6.3° B ≈ 145.7°



29) Impossible triangle.

Problem Set #3

1) Problems 23, 24 – SSS
   Problem 25 – ASA
   Problem 26 – SAS
   Problems 27 – 29 – SSA
2) The sum of the lengths of two sides is less than the third side.
3) The two given angles add up to 180° or more.
4) An SAS construction will never produce an impossible triangle as long as the given angle is less than 180°.
5) The given angle is obtuse and the given side adjacent to the angle is longer than the second given side.
6) The ratio of any two corresponding sides is equal to the ratio of any other two corresponding sides.
7) $x = 26$ cm
8) $x = \frac{35}{3} = 11\frac{2}{3}$ cm
9) – 12) Answers may vary.
13) $180(n – 2)$
14) $x = 113°$
15) $x = 125°$, $y = 75°$
16) $w = 39°$, $x = 90°$, $y = 39°$
    $z = 51°$
17) $x = 365$ m
18) $x = \sqrt{74} \approx 8.6$ in.
19) $x = 2\sqrt{6}$
20) $x = 6$, $y = 6\sqrt{2}$
21) $x = 15$ ft.
22) $x = \frac{60}{7} = 8\frac{4}{7}$ ft.

Problem Set #4

1) $3\sqrt{2}$
2) $5\sqrt{2}$
3) $10\sqrt{2}$
4) $\sqrt{2}$
5) $\sqrt{2}$
6) $\sqrt{2}$
7) The ratio of the hypotenuse to the leg of any 45-45-90 triangle is $\sqrt{2}$.
8) $7\sqrt{2}$
9) $\frac{9\sqrt{2}}{2}$
10) $x = 116°$, $y = 116°$, $z = 64°$
11) $x = 194$ ft.
12) $x = \sqrt{13} \approx 3.61$ in.
13) $\sqrt{5} \approx 2.24$ cm
14) $x = \frac{20}{3} = 6\frac{2}{3}$ cm
15) $x = 15$ cm
16) $y = \frac{32}{17} = 1\frac{15}{17}$ m,
    $x = \frac{60}{17} = 3\frac{9}{17}$ m
17) $x = 6\sqrt{2}$
18) $x = 3\sqrt{2}$
19) $3\sqrt{2}$
20) $\frac{7\sqrt{3}}{3}$
21) $6\sqrt{2}$
22) $\frac{4\sqrt{15}}{5}$
23) – 31) See the *High School Sourcebook* for directions on how to complete these constructions.

Problem Set #5

1) Short leg = 3, long leg = $3\sqrt{3}$
2) Short leg = 4, long leg = $4\sqrt{3}$
3) Short leg = $\frac{11}{2}$,
   long leg = $\frac{11\sqrt{3}}{2}$

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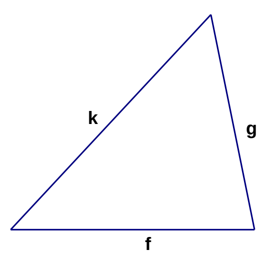
4) $\sqrt{3}$
5) 2 : 1
6) 2 : $\sqrt{3}$
7) $\sqrt{3}$: 1
8) 2 : $\sqrt{3}$
9) 2 : 1
10) 2 : $\sqrt{3}$
11) All three triangles are similar, and therefore the ratios are always equal.
12) 1 : $\sqrt{3}$: 2
13) $13\sqrt{3}$
14) $5\sqrt{3}$

15)



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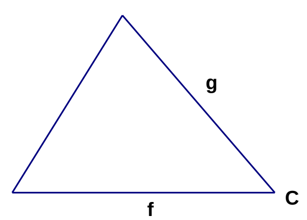
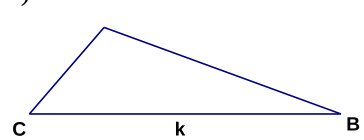


17)



18) $x = \frac{21}{4} = 5\frac{1}{4}$ ft.

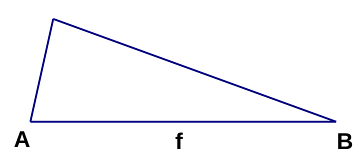
19) $x = \frac{64}{13} = 4\frac{12}{13}$ m

20) $x = \sqrt{7}$, $y = \frac{3\sqrt{14}}{5}$, $z = \frac{7}{5} = 1\frac{2}{5}$

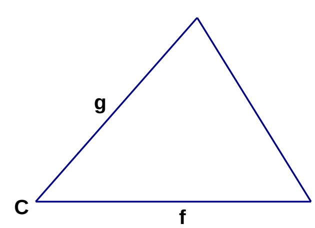
21) $x = 78°$
22) $x = 38°$
23) $x = 95°$, $y = 130°$
24) $x = 93°$
25) $x = 46°$, $y = 46°$
26) $x = \frac{9}{2} = 4\frac{1}{2}$, $y = \frac{9\sqrt{3}}{2}$
27) $x = 20$
28) $x = 8\sqrt{2}$
29) $x = 60$ m
30) $x = 5\sqrt{3}$, $y = 10$
31) $x = 5$, $y = 10$
32) $x = \sqrt{6}$
33) $\frac{15\sqrt{2}}{2}$
34) $x = \frac{5\sqrt{3}}{3}$, $y = \frac{10\sqrt{3}}{3}$
35) Definitely.
36) Not.

Problem Set #6

1) ASA.



2) SAS.



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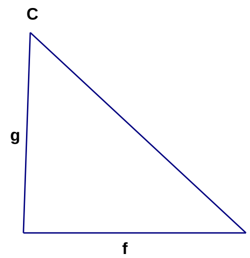
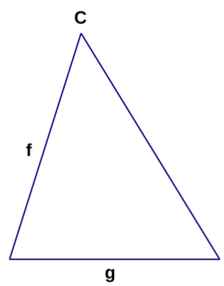
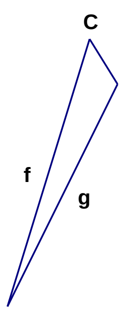
3) SSA.



4) SSA – Two possible constructions:



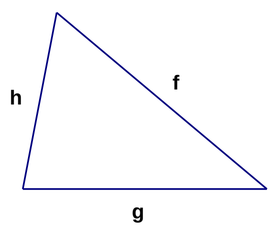


5) SSA – Not a possible triangle.
6) See answers 1 – 5.
7) See Problem Set #3 – Problems 2 – 5
8) If two sides and an angle between them in one triangle are congruent to two sides and an angle between them in another triangle, then the two triangles must be congruent.
9) If two angles and a side between them in one triangle are congruent to two angles and a side between them in another triangle, then the two triangles must be congruent.
10) If two angles and a side not between them in one triangle are congruent to two angles and a side not between them in another triangle, then the two triangles must be congruent.
11) In AAS, the known side is not between the two known angles. In ASA, the known side is between the two known angles. Essentially they are the same theorem because once two angles in a triangle are known, we can immediately find the third angle.
12) See previous answer.
13) Definitely. ASA
14) Definitely. SAS
15) Not definitely. SSA
16) Definitely. HL
17) $x = 44°$
18) $x = 98°$, $y = 40°$, $z = 122°$
19) $x = 111°$
20) $x = 128°$, $y = 147°$
21) $x = \frac{7}{3} = 2\frac{1}{3}$
22) $x = \frac{40}{3} = 13\frac{1}{3}$, $y = \frac{50}{3} = 16\frac{2}{3}$
23) $x = 6$, $y = 6\sqrt{3}$
24) $x = 28$, $y = 14\sqrt{3}$

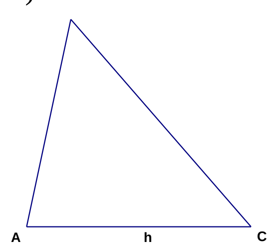
25) $x = 35\sqrt{2}$
26) $x = 2\sqrt{3}$, $y = 4\sqrt{3}$
27) $x = 6$, $y = 4\sqrt{3}$
28) $x = 5\sqrt{2}$
29) $x = 6\sqrt{21} \approx 27.5$ m
30) $x = 3\sqrt{14}$, $y = \frac{25}{3} = 8\frac{1}{3}$,

$z = \frac{5\sqrt{70}}{3}$

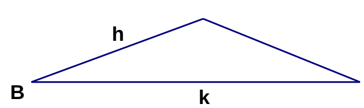
31) Definitely.
32) Definitely.
33) Not.
34) Definitely.
35)



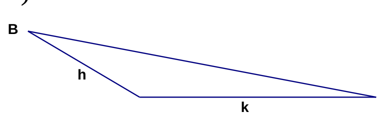
36) Not possible.
37)



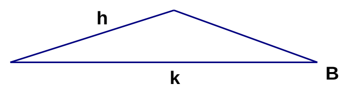
38)

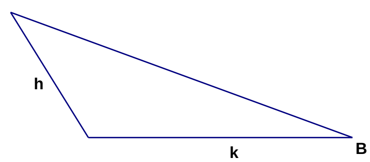


39)



40) 2 possible constructions:



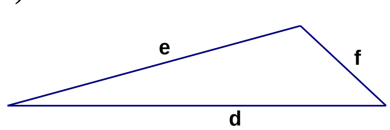


41) Not possible.

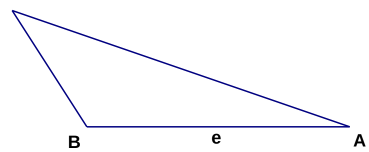
Problem Set #7

1) SSA is ambiguous. Having two sides and an angle not between them in one triangle congruent to two sides and one angle not between them in another triangle does not necessarily make the two triangles congruent. There could be another triangle with the same properties.
2) See problem 40 from the previous problem set.
3) Pythagorean Theorem.
4) AA means that the third angle is congruent as well. Similar triangles are not usually congruent. In addition, if it is known that two AA triangles have at least one equal side, then the two triangles are congruent.
5) Answers may vary.
6) Answers may vary.
7) Answers may vary.
8) $x = \frac{96}{5} = 19\frac{1}{5}$
9) $x = \frac{15}{4} = 3\frac{3}{4}$

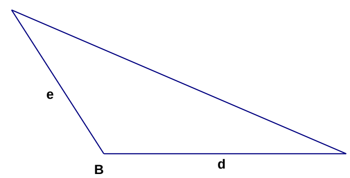
10) $x = \sqrt{6}$
11) $z = \sqrt{6}$
12)
  a) All three are similar.
  b) $z = \sqrt{xy}$
13) $H = \sqrt{15}$
14) $x = 4$, $y = 4\sqrt{3}$
15) $x = \frac{4\sqrt{3}}{3}$, $y = \frac{2\sqrt{3}}{3}$
16) $x = \sqrt{2}$, $y = 2\sqrt{2}$
17) $x = \frac{5\sqrt{2}}{2}$
18) $x = 5\sqrt{2}$, $y = 10$
19) $x = \frac{7\sqrt{3}}{2}$
20) $x = \frac{7\sqrt{3}}{3}$
21) $x = 6$
22) $a = \sqrt{10}$, $b = \sqrt{35}$, $c = \sqrt{14}$
23) $x = 10$
24) $a = \sqrt{13}$, $b = \frac{36\sqrt{13}}{13}$,
  $c = \frac{42\sqrt{13}}{13}$
25) $x = \frac{110}{7} = 15\frac{5}{7}$
26) $x = \frac{21}{4} = 5\frac{1}{4}$
27) $x = \frac{10\sqrt{3}}{3}$, $y = \frac{2\sqrt{111}}{3}$,
  $z = \sqrt{111}$
28)



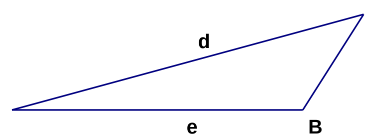
29)



30)



31)



32) Not possible.

Problem Set #8

1) Definitely. SAS
2) Definitely. AAS
3) Not definitely. SSA
4) Definitely. AA
5) Definitely. AA
6) Definitely. Ratios of all corresponding sides are equal.
7) Not definitely.
8) Definitely. Ratios of all corresponding sides are equal.
9) Definitely similar. SAS and ratios of corresponding sides are equal.
10) Not definitely.
11) 10

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12) $\frac{5}{2} = 2\frac{1}{2}$

13) $12\sqrt{3}$

14) $\frac{7\sqrt{3}}{3}$

15) $\frac{26\sqrt{3}}{3}$

16) $3\sqrt{3}$

17) $4\sqrt{2}$

18) $\frac{9\sqrt{2}}{2}$

19) $6\sqrt{3}$

20) $2\sqrt{3}$

21) $x = 3\sqrt{2}$, $y = 3$

22) $x = 8\sqrt{3}$, $y = 8$

23) $x = \frac{34\sqrt{3}}{3}$, $y = \frac{17\sqrt{3}}{3}$

24) $x = \sqrt{15}$

25) $x = \frac{33}{2} = 16\frac{1}{2}$ ft.

26) $x = \frac{5\sqrt{2}}{2}$, $y = \frac{\sqrt{194}}{2}$, $z = \sqrt{97}$

27) $D = \sqrt{66}$

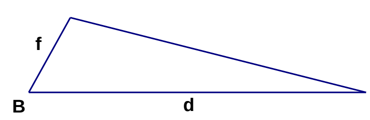
28) $x = \frac{9}{2} = 4\frac{1}{2}$

29) $Q = \sqrt{15}$, $P = 2\sqrt{10}$,
$W = 2\sqrt{6}$

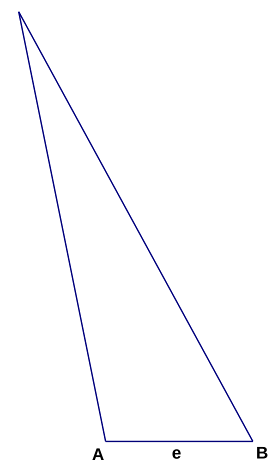
30) Q = 133°, W = 140°,
X = 92°, Y = 161°, Z = 42°

31) Not possible

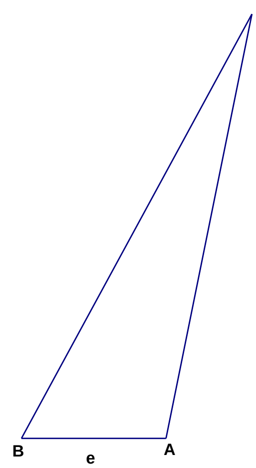
32)



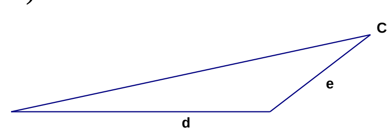
33)



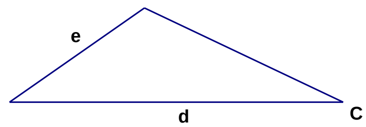
34) (Same as 33)

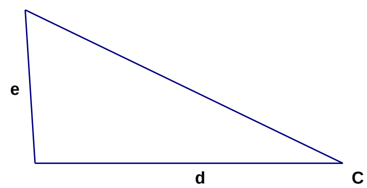


35)



36) Two possible constructions:

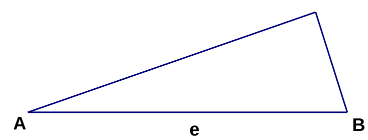




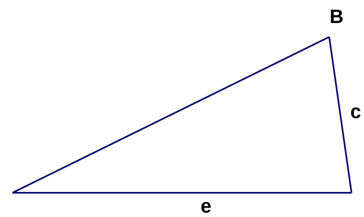
37) Not Possible

Problem Set #9

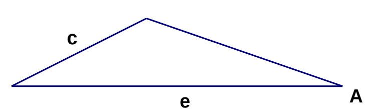
1) Definitely. AA.
2) Not. Ratios of corresponding sides are not equal.
3) Definitely. SAS and ratios of corresponding sides are equal.
4) Not definitely. SSA.
5) Definitely. HL.
6) Not definitely. Using the Pythagorean Theorem, the ratios of the corresponding sides are not equal.
7) $x = 123$ ft.
8) $x = 10$
9) $x = 78$ m
10) $x = \frac{21}{2} = 10\frac{1}{2}$
11) $x = 3\sqrt{2}$, $y = 6$
12) $x = \frac{3}{2} = 1\frac{1}{2}$, $y = \frac{3\sqrt{3}}{2}$
13) $x = 6\sqrt{3}$, $y = 12$
14) $x = \frac{4\sqrt{15}}{3}$
15) $x = \frac{4\sqrt{5}}{5}$, $y = \frac{2\sqrt{10}}{5}$
16) $x = \sqrt{11}$
17) $x = 6$, $y = 2\sqrt{3}$
18) $z = 2\sqrt{15}$, $D = 4\sqrt{10}$, $c = 4\sqrt{6}$
19) $x = \frac{50}{3} = 16\frac{2}{3}$, $D = \frac{10\sqrt{34}}{3}$, $C = 2\sqrt{34}$
20) $Z = 8$, $x = \frac{32}{3} = 10\frac{2}{3}$, $D = \frac{40}{3} = 13\frac{1}{3}$
21)

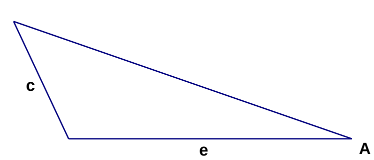


22)



23) Not possible.
24) Two possible constructions:



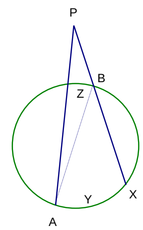


25) – 30) Constructions.

Circle Geometry ANSWERS

Group Worksheet

1) Parallel chords in a circle intercept equal arcs.
2) Two chords of equal length in a circle subtend equal arcs. The longest chord in a circle is the diameter which always intercepts equal arcs. Likewise with chords that are the hypotenuses of 45-45-90 triangles, etc.
3) $C = \frac{1}{2}(A + B)$
4)
   a) The measure of an angle is $\frac{1}{2}$ the measure of the arc it subtends.
   b) Answers may vary.
   c) The inscribed angle is 90°.
   d) *Thales' Theorem*.
5) The angle formed by the intersection of the chord and the tangent is half of the arc it intercepts. We can use the *Inscribed Angle Theorem* to see this is true.
6) The tangent line and the diameter are perpendicular. We can use the *Chord-Tangent Theorem* to see this is true.
7) The outside angle is equal to half the difference of the measures of the two arcs it subtends. Let P be the outside angle, Z the intercepted arc closest to P and Y the other intercepted arc:



$P = \frac{1}{2}(Y - Z)$

We can be sure this is true using ΔABP and the *Inscribed Angle Theorem*:

$\angle PAB = \frac{1}{2}Z$

$\angle PBA = 180 - \frac{1}{2}Y$

$\therefore P =$

$180 - (\frac{1}{2}Z + 180 - \frac{1}{2}Y)$

thus $P = \frac{1}{2}(Y - Z)$

8) The two tangents are equal in length from the point of intersection to the point of tangency.
9) The opposite angles of the quadrilateral are supplementary. We can be sure this is true using the *Inscribed Angle Theorem.*
10) $AP \cdot BP = CP \cdot DP$
11) $PB \cdot PA = PD \cdot PC$
    We can be sure this is true by using the *Chord Segment Theorem.*
12) $PB \cdot PA = (PC)^2$
13) In a circumscribed quadrilateral, the sum of a pair of opposite sides is equal to the sum of the other pair of opposite sides.

Circle Geometry ANSWERS

Problem Set #1

1) $x = 31°$. *Inscribed Angle Theorem.*.
2) $x = 48°$. *Intersection Chord Theorem.*
3) $x = 90°$. *Inscribed Angle Theorem* (or in this case *Thales' Theorem*).
4) $x = 15°$. *Parallel Chord Theorem.*
5) $x = 120°$. Properties of a circle.
6) $x = 32°$. *Outside Angle Theorem.*
7) $x = 65°$. *Chord-Tangent Theorem.*
8) $x = 4$. *Intersecting Tangent Theorem.*
9) $x = 90°$. *Diameter-Tangent Theorem.*
10) $x = 93°$. *Inscribed Quadrilateral Theorem.*

Problem Set #2

1) $x = 70°$
2) $x = 70°$
3) $x = 55°$
4) $x = 58°$
5) $x = 44°$
6) $x = 108°$
7) $x = 104°$, $y = 52°$, $z = 152°$
8) $x = 103°$, $y = 70°$
9) $x = 64°$
10) $w = 132°$, $x = 42°$, $y = 66°$, $z = 144°$
11) $w = 38°$, $x = 26°$, $y = 85°$, $z = 61.5°$

Problem Set #3

1) $x = 124°$
2) $x = 59°$
3) $x = 57.5°$
4) $x = 3\frac{1}{4}$
5) $x = 30°$
6) $x = 138°$
7) $x = 20°$, $y = 82°$
8) $x = 10$
9) $w = 55°$, $x = 117.5°$, .$y = 62.5°$
10) $x = 26°$, $y = 156°$, $z = 52°$
11) $x = 12$

Problem Set #4

1) $x = 23°$
2) $x = 68°$
3) $x = 62.5°$
4) $x = 139°$
5) $x = 84°$
6) $x = 50°$, $y = 105°$
7) $x = 32°$
8) $x = 52°$, $y = 52°$
9) $x = 132°$
10) $x = 6$
11) $x = \frac{416}{19} = 21\frac{17}{19} \approx 21.9$

Circle Geometry ANSWERS

Problem Set #5

1) $x = 105°$, $y = 75°$
2) $x = 31°$
3) $x = 144°$
4) $x = 116°$
5) $x = 7$
6) $w = 59°$, $x = 121°$, $y = 121°$
7) $x = 64°$
8) $x = \frac{9}{4} = 2\frac{1}{4}$
9) $x = 24$
10) $x = 20$
11) $x = \frac{11}{5} = 2\frac{1}{5}$
12) Q = 106°, $w = 44°$, $x = 88°$, $y = 96°$, $z = 44°$
13) $w = 32°$, $x = 26°$, $y = 34°$

Problem Set #6

1) The length of the tangent segment is $4\sqrt{2}$ cm. In general, the tangent segment length would be the product of length of the external or internal portion of the secant segment and $\sqrt{2}$.
2) This problem requires two equations. $x = 40.5°$.
3) $x = 190°$
4) $x = 124°$
5) $x = 48°$, $y = 24°$, $z = 24°$
6) $x = 146°$, $y = 143°$
7) $w = 110°$, $x = 110°$, $y = 70°$, $z = 56°$
8) $x = 40°$
9) $x = 9$
10) This requires a factorable quadratic. Since -10 is not a possible answer, $x = 4$.
11) This requires using the quadratic formula: $x = -4 + \sqrt{65} \approx 4.062$
12) $x = \frac{27}{5} = 5\frac{2}{5}$
13) $w = 35°$, $x = 35°$, $y = 23°$, $z = 44°$

Problem Set #7

1) $x = 22.5°$, $y = 85°$
2) $x = 115°$
3) $x = 55°$
4) $x = 32°$
5) $x = 115°$, $y = 90°$, $z = 70°$
6) Q = 80°, $w = 40°$, $x = 120°$, $y = 40°$, $z = 40°$
7) $w = 34°$, $x = 11°$, $y = 144°$, $z = 52°$
8) $x = 58°$, $y = 34°$
9) $x = 16$
10) $x = \frac{-7+\sqrt{229}}{2} \approx 4.066$
11) $x = \sqrt{66} \approx 8.124$
12) Use properties of similar triangles to create two equations. Then use substitution to create a quadratic. $x = 2$

Circle Geometry ANSWERS

1) 4 possible centers:
- Circumcenter – Intersection of the perpendicular bisectors of each side.
- Incenter – Intersection of lines bisecting each angle.
- Centroid – Intersection of the lines connecting vertices with midpoints of opposite sides.
- Orthocenter – Intersection of the three altitudes.

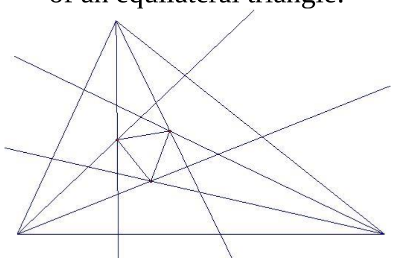
2) See previous problem.

3)
   a) In an equilateral triangle.
   b) In an isosceles triangle.
   c) The circumcenter, orthocenter and centroid <u>always</u> fall in a line.
   d) The orthocenter and circumcenter in obtuse triangles.
   e) The orthocenter and circumcenter in a right triangle.

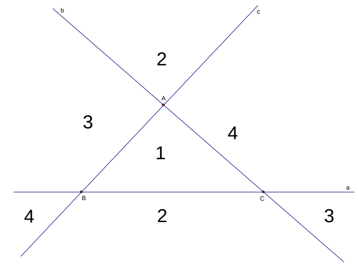
4) Answers may vary.

5)
   a) Infinity.
   b) When the triangle is infinitely thin so it is essentially a line.

6) The three points are vertices of an equilateral triangle:

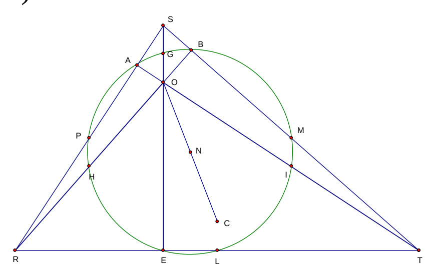


7) The point must travel from line *a* to point A through infinity.



8)
   a) 4
   b) 1
   c) 1
   d) 1

9)



10) The 9-point circle is tangent to the other four circles.

11) Points X, Y and Z are collinear.

12) The resulting curve is called a hypocycloid, which is tangent to both the nine-point circle and the original triangle, and has the reverse orientation from its Morley's Triangle.