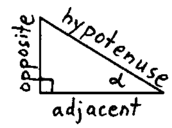
11^{th} Grade Assignment – Week #6

Individual Work

- Before the end of this week, take the **Cartesian Geometry – Part I** test that is found at the end of this document. You should take the test at home, supervised by your parent. You should not use notes for the test. If it is possible for you, try not to use a calculator. Send the test to your tutor once you have completed it.
- Do **Problem Sets #1 and #2** from the unit **Trigonometry – Part II.**

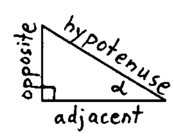
Group Assignment:

For Tuesday.



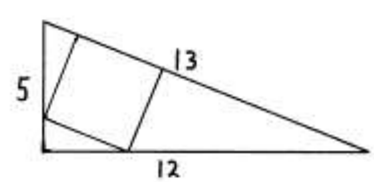
1) Make sure everyone in the group understands everything below.
   - $sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan(\alpha) = \frac{\text{opposite}}{\text{adjacent}}$
   - $\sin(\alpha)$ answers the question: "The opposite leg is how much of the hypotenuse?"
   - $\cos(\alpha)$ answers the question: "The adjacent leg is how much of the hypotenuse?"
   - $\tan(\alpha)$ answers the question: "The opposite leg is how much of the adjacent leg?"
2) Write down the two special triangles – 30-60-90 and 45-45-90, including the ratios of the sides.
3) Fill out the below table. For irrational answers, give the values both in square root (exact) form, and as decimal approximations.

θ	**sin θ**	**cos θ**	**tan θ**
0°			
30°			
45°			
60°			
90°			



4) If you take a triangle with sides of 6, 8, and 10, and then divide the whole triangle by 2, then the new triangle will have sides of length 3, 4, and 5. Similarly, take the triangle shown here with sides "opp", "adj", and "hyp", and divide everything by "hyp". What do you end up with? Why is this special? What formula do you end up with if you apply the Pythagorean Theorem to this new triangle?
5) Using the view of sine (that I gave in the lecture) where you have an inscribed angle inside a circle, find the exact values of sin(90°), sin(120°), sin(135°), sin(150°), and sin(180°). (Hint: The Inscribed Angle Theorem (from last week's assignment) may be helpful here.) What identity (law) is reflected in what you did above?
6) Find x: a) sin(30°) = cos(x) b) sin(17°) = cos(x) c) cos(23°) = sin(x) What identity (law) is reflected in what you did above?
7) What is $\frac{\sin(\alpha)}{\cos(\alpha)}$ always equal to?

For Thursday.



8) The figure shown here is a square inscribed inside a 5-12-13 right triangle. Find the area of the square.

Cartesian Geometry I Test

All Problems are worth 4 points, unless otherwise indicated.

1) Give the equation of each line, given below.

a)

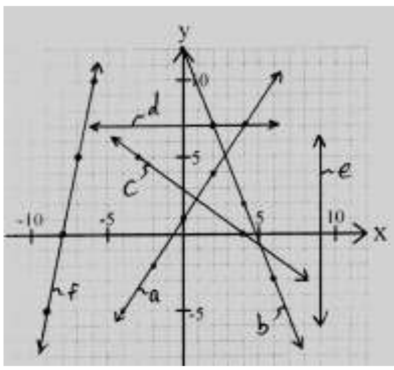
b)

c)

d)

e)

f)



2) Find the exact point of intersection of the lines 1a and 1c, given above. (You may use any method.)

Use the graphs below to graph each of the following equations. Be sure to label each one.

a) $y = \frac{2}{3}x - 4$

b) $y = -x + 1$

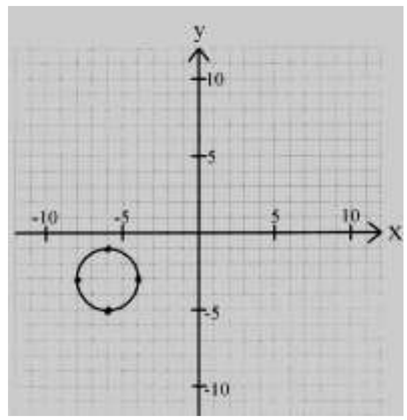
c) $y = -3x$

d) $2x + y = 6$

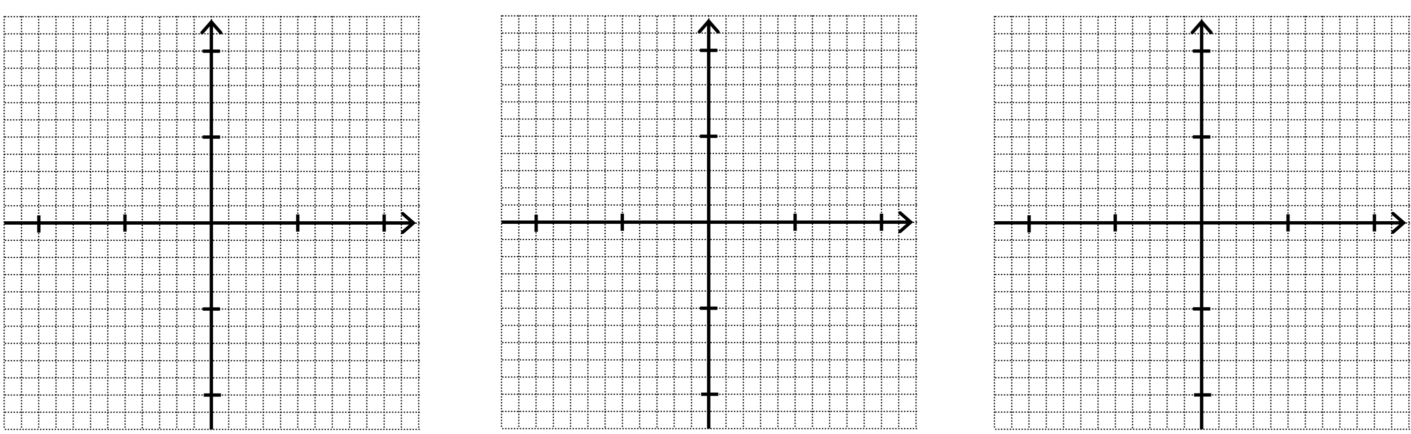
e) $4x - 5y = 15$

f) $x = y^2 - 6$

4) Give two solutions to the equation of the below graph.



3)



5) Find the (exact!) common solution of these two equations. (You must use the method of substitution.)

$$3x + 5y = 1$$
$$2x - y = 5$$

6) Find the (exact!) common solution of these two equations. (You must use the method of linear combination.)

$$2x + 5y = 7$$
$$3x - 2y = -18$$

7)

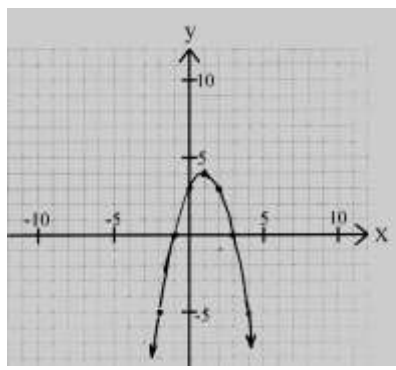
Give the equation of the line that…

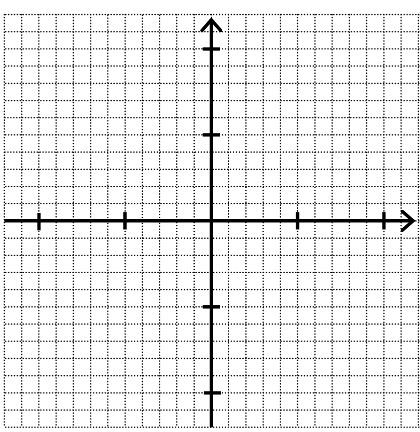
a) Has a slope of –3 and passes through the point (2,–7).

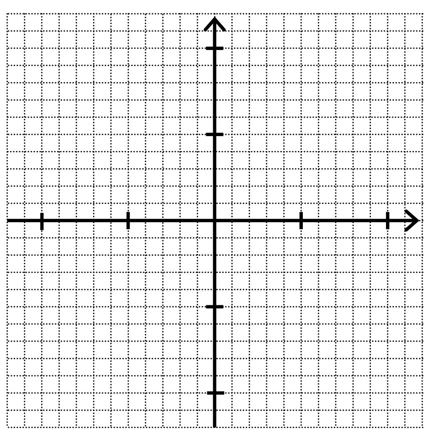
b) Passes through the points (3,4) and (6,9).

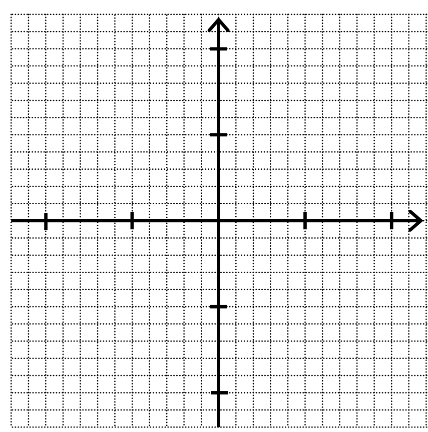
c) Passes through the point (1,–7) and is perpendicular to the line $y = \frac{2}{3}x - 245$

8) *Challenge!* (Do only if you have extra time.) (2 points extra credit)
Give the equation of the below graph.









Trigonometry – Part II

Background

A Summary of Last Year

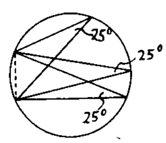
The beginning of our study of trigonometry in eleventh grade is based upon the *Introduction to Trigonometry* unit (from the previous workbook). I will now summarize and review last year's study of trigonometry.

It began with a look at how Ptolemy of Alexandria (ca. 150A.D) had developed the first trig table. Ptolemy's table was based upon the length of chords in a circle of fixed size. Our goal was to create a similar table, but instead based upon the modern trigonometric function called *sine*, which then became the foundation of our studies of trigonometry.

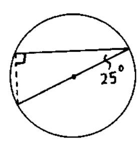
The Idea of *Sine*: Given α as the degree measure of an *inscribed angle* in a circle, the value of $sin(\alpha)$ indicates the length of the resulting chord as a proportion of the length of the circle's diameter.

- $sin(\alpha)$ essentially answers the question: "*The chord is what proportion of the diameter*?"
- $sin(25°) \approx 0.423$ means that if the inscribed angle is 25°, then the length of the chord that subtends it would be approximately 0.423 times as long as the circle's diameter.

Interestingly, it didn't matter where the vertex of the inscribed angle lied on the circle – the length of the chord would be the same.

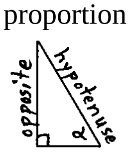


Furthermore, we could choose to conveniently place the vertex of the angle in a place that would create a right triangle. In this case, the hypotenuse of the triangle would be a diameter of the circle.



We can now ignore the circle altogether, which brings us to a picture of *sine* that is consistent with how most people view it today:

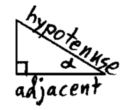
- $sin(\alpha)$ essentially answers the question: "The opposite side is what proportion of the hypotenuse?"
- $sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}}$



The Idea of *Cosine*:

We then saw that we frequently needed to find the *sine* of the other angle in the right triangle (the complementary angle); this is how the idea of *cosine* emerged.

- $\cos(\alpha)$ essentially answers the question: "The adjacent leg is what proportion of the hypotenuse?"
- $\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}}$



Notice that although *sine* can be thought of independently of a right triangle, *cosine* can only be thought of (for the time being) as a relationship within a right triangle.

Trigonometric Identities

In the process of following Ptolemy's path and filling out our trigonometric table, we encountered several formulas, usually referred to as *trigonometric identities*. See if you can remember what each one means and why it was useful.

- $sin(180°-\alpha) = sin(\alpha)$
- $sin(90°-\alpha) = cos(\alpha)$
- $sin^2\alpha + cos^2\alpha = 1$
- $\frac{a}{b} = \frac{sin\,A}{sin\,B}$
- $sin(½\alpha) = \sqrt{½ - ½cos\,\alpha}$
- $sin(\beta-\alpha) =$ $sin(\beta)cos(\alpha) - sin(\alpha)cos(\beta)$
- $cos(\alpha+\beta) =$ $cos(\alpha)cos(\beta) - sin(\alpha)sin(\beta)$

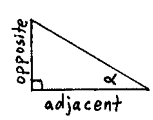
In this unit, we will only be using the first four of these identities.

Some New Ideas

Tangent – the next Trig Function

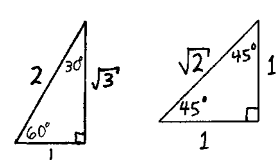
The third trigonometric function, *tangent*, once again involves the ratio of the lengths of two sides of a right triangle.

- $\tan(\alpha)$ essentially answers the question: "The opposite leg is what proportion of the adjacent leg?"
- $\tan(\alpha) = \frac{\text{opposite}}{\text{adjacent}}$



Special Triangles

The following two triangles appear frequently in trigonometry and the ratio of the lengths of their sides should be memorized.



Problem Set #1

1) <u>The Basic Trig Facts.</u>
It will help to use the *special triangles*, as shown above. Give your answers both as fractions and as decimal approximations. (Don't use the trig buttons on your calculator.)

a) *cos*(45°)

b) *sin*(45°)

c) *tan*(45°)

d) *cos*(60°)

e) *sin*(60°)

f) *tan*(60°)

g) *cos*(30°)

h) *sin*(30°)

i) *tan*(30°)

j) *cos*(0°)

k) *sin*(0°)

l) *tan*(0°)

m) *cos*(90°)

n) *sin*(90°)

o) *tan*(90°)

2) For each given angle do the following:
- Draw a freehand right triangle that has the given angle.
- Give a decimal estimate of the *sin*, *cos* and *tan* of this angle.
- Use the trig buttons on your calculator to check your answer, and calculate the percent error of your answer.

a) 10°

b) 55°

c) 35°

d) 75°

e) 15°

Problem Set #2

1) For each problem, give a decimal estimate, then use your calculator to check it.

a) $\cos(80°)$

b) $\sin(80°)$

c) $\tan(80°)$

d) $\cos(25°)$

e) $\sin(38°)$

f) $\tan(8°)$

2) For each problem, you are given the three sides of a right triangle and one of the angles. Give the *cos*, *sin* and *tan* of the given angle. (Give answers as fractions.)

a) Sides = 3, 4, 5;
angle ≈ 36.9°

b) Sides = 30, 40, 50;
angle ≈ 36.9°

c) Sides = 6, 8, 10;
angle ≈ 36.9°

d) Sides = 5, 12, 13;
angle ≈ 67.4°

e) Sides = 10, 24, 26;
angle ≈ 67.4°

f) Sides = 28, 45, 53;
angle ≈ 31.9°

g) Sides = 28, 45, 53;
angle ≈ 58.1°

3) What principles are shown by the preceding problems?

4) Give an explanation of why each of the following identities is valid.

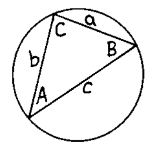
a) $\sin(180°-\alpha) = \sin(\alpha)$

b) $\sin(90°-\alpha) = \cos(\alpha)$

c) $\sin^2\alpha + \cos^2\alpha = 1$

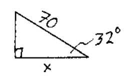
d) $\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$

e) $\frac{a}{b} = \frac{\sin A}{\sin B}$

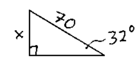


5) Find the variable indicated.

a)



b)



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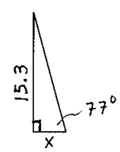
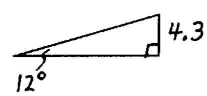


6) Find all the missing sides and angles.



7) Work on memorizing the values for the *Basic Trig Facts* (as given on problem #1 from Problem Set #1). It may be best to make flashcards.