11th Grade Assignment – Week #4

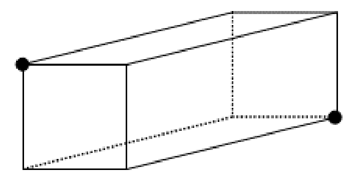
Note: There is graph paper on the next page. You may wish to print several copies of that one page.

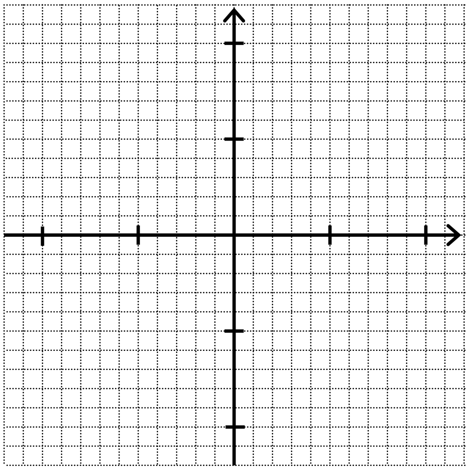
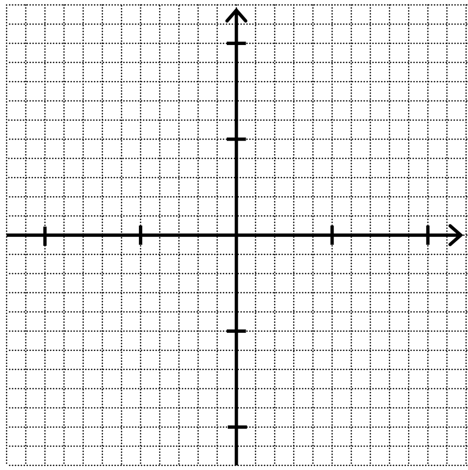
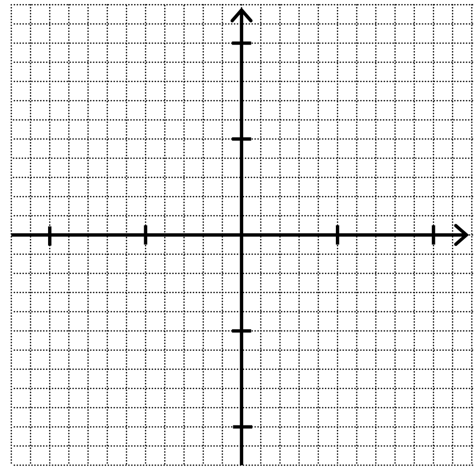
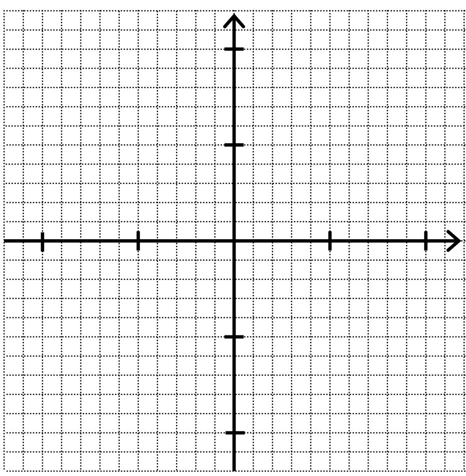
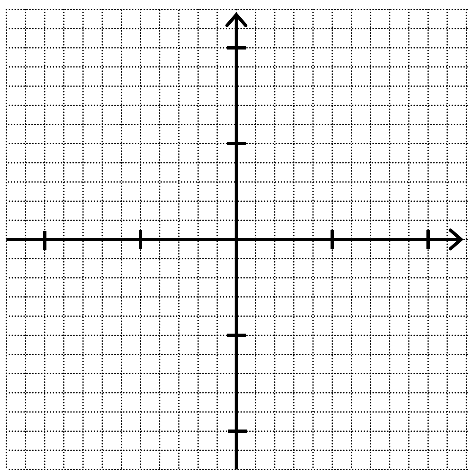
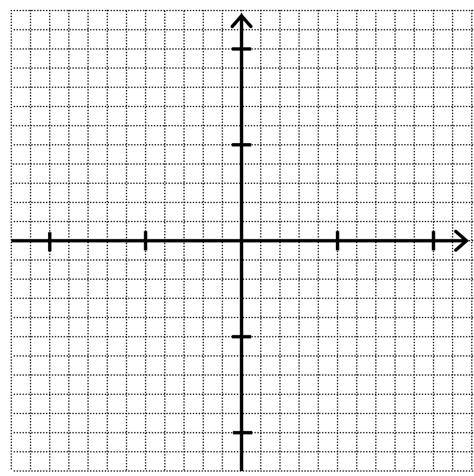
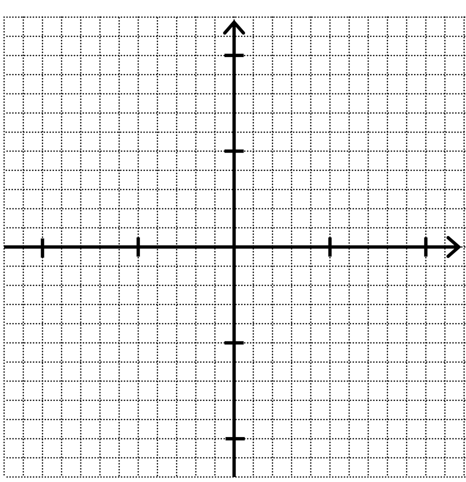
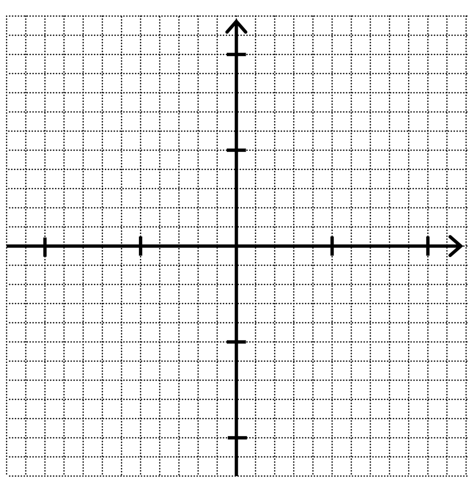
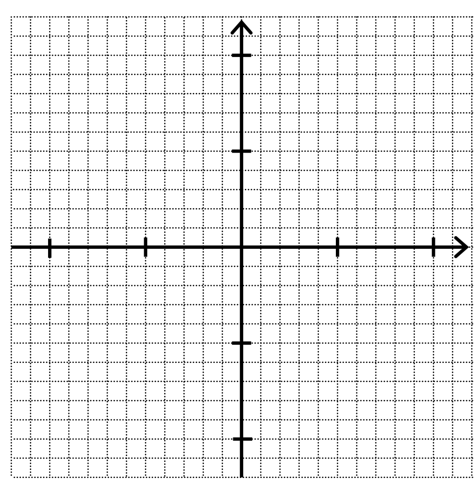
Individual Work

- Do as much as you can with the problems on **Problem Sets #1-3**, from the workbook unit, **Cartesian Geometry – Part I.**
  Important Note: Because this material is so fundamental for all future math studies, even if you are already familiar with this material, it is important that you carefully look through all the problems (you don't have to do them all), to make sure that you really know how to do all of them.

Group Assignment: *For Tuesday and Thursday*

- Help each other out with the problems from the Individual Work (above).
  Especially focus on the more challenging problems.
  Again, this work is important; make sure everyone understands!
- **The Two-Ant Puzzle – Part I.**
  Two ants (one male, one female) are at diametrically opposite corners inside a box that measures 24x24x60 cm. What would be the length of the shortest path walking from one ant to the other?



Cartesian Geometry – Part I

Problem Set #1

In 1637, René Descartes published a book with the impressive title *Discourse on the Method of Rightly Conducting One's Reason and Searching for the Truth in the Sciences*, which today is seen as a work of major importance in the fields of philosophy and in general science. The book also included an appendix on geometry where he showed how his new method of conducting science could be used to develop a new way of solving geometric problems. Before Descartes, geometry and algebra were separate subjects.

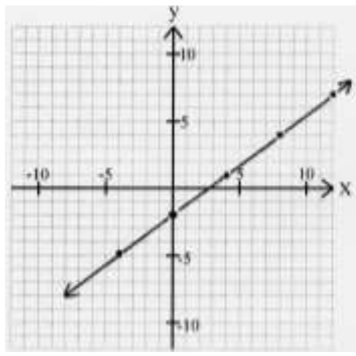
What Descartes actually did was to take a geometry problem (the Pappus problem) and expressed it as an equation. Descartes' seed idea (and Fermat came up with similar ideas at about the same time) was then further developed over a period of time, and has been tremendously influential in the world of science and mathematics. Modern Cartesian geometry (which is also referred to as coordinate geometry or analytical geometry) allows us to take an equation and express it as geometry; *it allows us to visualize algebra*.

To graph an equation, we simply follow Descartes' words: "We may give any value we please to either x or y, and find the value of the other from the equation", in order to find several solutions. Then you plot the solutions to each equation on a Cartesian graph. For many equations, you have to carefully choose the values for x or y so that the points you plot fit reasonably on the graph.

<u>Example</u>: $y = \frac{3}{4}x - 2$
<u>Solution</u>: It is easiest, perhaps, to plug in multiples of 4 into x. (Although we could have instead chosen fairly random values for x.)
This leads to the following table:

x	–4	0	4	8	12
y	–5	–2	1	4	7

Note that for every step of the four that the x values take, the y values take a step of three. (We will see later why this is important.) Lastly, we simply need to graph the solutions that we have found, and we see that the result is the straight line shown here:



Graph each equation.

1) $x^2 + y^2 = 25$

2) $y = \frac{1}{2}x + 3$

3) $y = x^2 + 6x + 5$

4) $y = 2x^3 - 3x + 1$

5) $y = x^4 - 5x^2 + 4$

Problem Set #2

On the previous problem set we were able to graph equations making a table and then plotting points. While the method of making a table and plotting points can be reliable, it is time consuming and tedious.

You may have noticed on the last problem set that the equations without any exponents ended up having graphs that were straight lines. Such equations are called *linear equations*. Mathematicians are always searching for more efficient ways to do things; this problem set is focused on finding quicker ways to graph linear equations.

1) Graph each of the following on the same graph by making a table and then plotting points.

a) $y = 2x + 1$

b) $y = 2x - 3$

c) $y = 2x + 4$

d) $y = 2x - 6$

2) With the above equations, what does the number at the end of the equation tell you?

3) Graph each of the following on the same graph by making a table and then plotting points.

a) $y = 2x + 1$

b) $y = \frac{2}{5}x + 1$

c) $y = \frac{5}{2}x + 1$

d) $y = -\frac{5}{2}x + 1$

e) $y = -\frac{3}{5}x + 1$

4) In each of the above equations, what does the number before the "x" tell you?

5) Now, given what you have learned above, graph each of the following without making a table .

a) $y = {}^{3}/_{2}\, x - 4$

b) $y = -\, {}^{1}/_{3}\, x - 2$

c) $y = -\,3x$

Two Forms

In general, there are two common forms for expressing linear equations. One is called *standard form*, where there are no fractions and the x's and y's are both on the left side, such as:

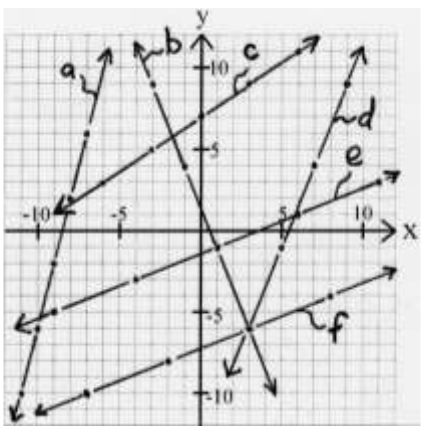
$4x + 3y = 15$

If we now solve this equation for y, then we get *slope-intercept form*, which for the above equation is: $y = -\frac{4}{3}x + 5$

Problem Set #3

On the previous problem we saw how if we have an equation solved for y, then the number in front of the x (which is called the *slope*) tells us how steep the line is, and the constant at the end tells us where the line crosses the y-axis (and this is called the *y-intercept*, and it is where the value for x is equal to zero).

1) Give the slope of all of the lines below.



Questions about slope…

(It may be helpful to look at your answers to the previous problem.)

2) What does a negative or positive slope tell us about the direction of the line?

3) What is the slope of a line that is 45° (off horizontal)?

4) What can be said about the slope of a line that is less steep than 45°?

5) What can be said about the slope of a line that is steeper than 45°?

6) What can be said about the slopes of two lines that are parallel with each other?

7) What can be said about the slopes of two lines that are perpendicular to each other?

8) What is the slope of a line that is horizontal?

9) What is the slope of a line that is vertical?

10) Graph the following equations.

a) $y = \frac{1}{3}x - 4$

b) $y = {}^{3}/_{2}\, x - 1$

c) $y = -\, {}^{3}/_{2}\, x + 1$

d) $y = \frac{3}{4}x + 2$

e) $y = \frac{1}{2}x$

f) $y = -5x$

g) $y = 2x - 5$

h) $y = -3x + 2$

i) $y + 3x = 2$

j) $4y - x = -8$

k) $y = 4$

l) $3x + 2y = 2$

11) The last equation above is the same as what other equation given further above?

12) Consider the equation $y = x^2 - 4x$.

a) Give three solutions to the equation.

b) What are all the possible values that x can have? (In other words, is there a limit to how big or small x can be?)

c) What are all the possible values that y can have? (In other words, is there a limit to how big or small y can be?)

d) Graph the equation.

e) Does graphing the equation give you any insights into the answers to part b and c?

13) Consider the equations

$4y + 3x = 6$

and $y - 2x = 7$

a) Give three solutions to each equation.

b) Find the common solution to the two equations by using algebra.

c) Graph each equation (on the same graph).

d) What is the common point on the graph?