11^{th} Grade Assignment – Week #3

Announcements:

- Try to read through Part IV of "Excerpts from Descartes's *Discourse on the Method*…" (given with last week's assignment) before your Tuesday group meeting.
- Note that there are two extra videos this week. (See below.)
- Here is Descartes's "seed" for what we now know as Cartesian Geometry (from p34 of section H):

  > "We may give any value we please to either x or y and find the value of the other from this equation…If then we should take successively an infinite number of values for the line y, we should obtain an infinite number of values for the line x, and therefore an infinity of different points, such as C, by means of which the required curve could be drawn."

Group Assignment:

For Tuesday

- Together, do your best to work through Problem #1from **Worksheet #3** (from the document *Descartes Worksheets*). This is the Pappus Problem
- After reading Part IV of "Excerpts from Descartes's *Discourse on the Method*…", do the following, together as a group:
  - Much of Part IV is focused on his argument for the existence of God. He gives more than one justification. Together discuss how he justifies God's existence, and write down a summary of each of his arguments.

For Thursday

- Questions for discussion:
  - Descartes's was able to unite two branches of mathematics: geometry and algebra. In what ways has this influenced the development of our world?
  - One of Descartes's central ideas was his precept that allows us to tackle large problems by breaking it into its parts and dealing separately with the parts. Discuss the positive and negative consequences of this "reductionist approach".
- Together, come up with rough sketches of what Problems #2-4 from **Worksheet #3** (from the document *Descartes Worksheets*) should look like. Then answer this question: How could you move the lines in order to create a parabola? (The answer to this question is shared in the below "Pappus Problem Solution" video.)

Individual Work

- Write about any topic or question that has interested you.
- Watch the extra video for the "Pappus Problem Solution": (JYMA – G11 – W03 – L3)
  Note that this is a video from a workshop for high school math teachers.
- *Extra Challenge!* Watch the extra video on the "Descartes's Section H": (JYMA – G11 – W03 – L4)
  Again, this is a video from a workshop for high school math teachers.
  Note that my notes for this are below.
  Your task is to them write a concise summary of what Descartes did in section H, in a manner so that someone with a good understanding math, but not present in this course, could understand it.

Section H of *La Géométrie*

The Key Ideas

- This is a summary of what Descartes did (as described in *La Géométrie*, section H) in his treatment of the Pappus problem. The whole section tells in detail how you can find one point, C, on the resulting curve.
- We begin with the idea that from point A (the origin) we can travel any given distance, x, along the line EG, and from there (point B) travel a certain distance, y, at the required angle (θ_3) until we intersect (at point C) the desired loci curve.
- It is important to note that if we imagine point C moving along the desired loci curve, then all of the triangles (see below) will retain their same shape, angles, and ratios, even though their size will change.
- The goal is to derive a formula that, given any x value, calculates y, or, given any value for y, calculates x.
- The four lines are placed on the page, and we are to find point C on the curve such that the distances to each line – measured at the required angles (θ_1, θ_2, θ_3, θ_4) – satisfy the equation **CB·CF = CH·CD**.

Triangle Ratios (Descartes has chosen to express all of his ratios in terms of z, which could be any number.)

Using ΔARB, let AB:BR = z:b.

$BR = \frac{b \cdot AB}{z}$ since AB = x we get $BR = \frac{b \cdot x}{z}$

Using ΔDRC, let CR:CD = z:c. $CD = \frac{c \cdot CR}{z}$

Using ΔESB, let EB:BS = z:d.

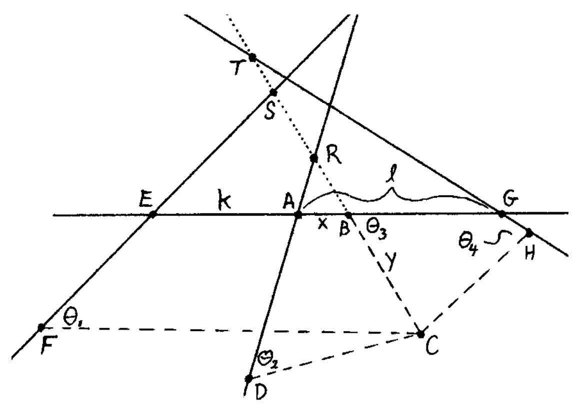
$BS = \frac{d \cdot EB}{z}$ since EB = k+x we get $BS = \frac{dk + dx}{z}$

Using ΔFSC, let CS:CF = z:e. $CF = \frac{e \cdot CS}{z}$

Using ΔBGT, let BG:BT = z:f.

$BT = \frac{f \cdot BG}{z}$ since BG = $\ell - x$ we get $BT = \frac{f\ell - fx}{z}$

Using ΔTCH, let TC:CH = z:g. $CH = \frac{g \cdot CT}{z}$



Deriving Formulas for the Four Distances (CB,CF, CH, CD)

- Calculating CB. $\boxed{CB = y}$ (The rest aren't that easy!)
- Calculating CD.
  CR = BR+BC. Given BC=y and $BR = \frac{b \cdot x}{z}$ (from above), we get $CR = \frac{b \cdot x}{z} + y$

  $CD = \frac{c \cdot CR}{z}$ (from above). Substituting, we get $CD = \frac{c \cdot \left(\frac{b \cdot x}{z} + y\right)}{z} \rightarrow CD = \frac{cy}{z} + \frac{bcx}{z^2}$ (the second c is missing in the translation) which becomes $\boxed{CD = \frac{czy + bcx}{z^2}}$
- Calculating CF.
  CS = BS + CB. Given CB = y and $BS = \frac{dk + dx}{z}$ (from above), we get $CS = \frac{dk + dx}{z} + y \rightarrow CS = \frac{dk + dx + zy}{z}$

  $CF = \frac{e \cdot CS}{z}$ (from above). Substituting, we get $CF = \frac{e \cdot \left(\frac{dk + dx + zy}{z}\right)}{z}$ which becomes $\boxed{CF = \frac{ezy + dek + dex}{z^2}}$
- Calculating CH.
  CT = BT + CB. Given CB = y and $BT = \frac{f\ell - fx}{z}$ (from above), we get $CT = \frac{f\ell - fx}{z} + y \rightarrow CT = \frac{zy + f\ell - fx}{z}$

  $CH = \frac{g \cdot CT}{z}$ (from above). Substituting, we get $CH = \frac{g \cdot \left(\frac{zy + f\ell - fx}{z}\right)}{z}$ which becomes $\boxed{CH = \frac{gzy + fg\ell - fgx}{z^2}}$

Section H of *La Géométrie* (continued)

- The first sentence at the top of page 34 restates the condition of the Pappus problem: CB·CF = CH·CD. This can therefore be expressed as:

$$y \cdot \left(\frac{ezy + dek + dex}{z^2}\right) = \left(\frac{gzy + fg\ell - fgx}{z^2}\right) \cdot \left(\frac{czy + bcx}{z^2}\right)$$

- In the first sentence on page 33, he says that with each of the above formulas x and y are the only variables, and all of the rest (c, b, z, e, d, e, g, f) are given, or easily calculable. Then he says (p34) that **we can assign a value to y, and because all the other variables except x are known, this equation can then be reduced to the form of $x^2 = \pm jx \pm k^2$**, which is something that he has shown us how to solve earlier. We have therefore achieved our goal of deriving a formula that calculates x, given any y value. (Likewise, we could randomly choose any value for x, and then calculate the corresponding value for y, which would yield a point on the curve.)

- The essence of what we know to be Cartesian geometry today is found in the two sentence on page 34:

  > "We may give any value we please to either x or y and find the value of the other from this equation…If then we should take successively an infinite number of values for the line y, we should obtain an infinite number of values for the line x, and therefore an infinity of different points, such as C, by means of which the required curve could be drawn."