10th Grade Assignment – Week #7

<u>Group Assignment</u>:

for Tuesday

- **<u>Euclid's Proof of I-47</u>** (the Pythagorean Theorem)
    - Justify the steps of Euclid's proof of I-47 (See the proof at the end of this document.) (Answers are at the end of next page – but don't look until you've really worked on it!)
    - Together, come up with a couple of sentences that summarize how Euclid proved I-47 (Pythag. Th.)

for Thursday

- **<u>Intersecting Chord Theorem</u>**

  Each person in the group should draw a circle with a radius of about 6cm, and mark its center. As I demonstrated in the lecture, you should then draw any two chords inside the circle so that they intersect. Label the acute angle formed by the intersecting chords as ∠C. Now label the arc subtended by ∠C as arc A, and the opposite arc as arc B. Use a <u>protractor</u> to measure the arcs A and B and ∠C. Now repeat the same process with two more circles, but by having each pair of chords intersect very differently. In the end, each group member should have three values for A, B, and C.

  Very Important: You must be sure that you use your protractor correctly to measure the angles, and that you carefully follow the directions as I explained in the lecture.

  Record everyone's results in a single table, with columns for A, B, and C, each measured in degrees. If you have 4 members in your group, there should be 12 rows of values.

  Now comes the hard part! By looking at the table, see if you can discover a math law or formula that relates A, B and C.

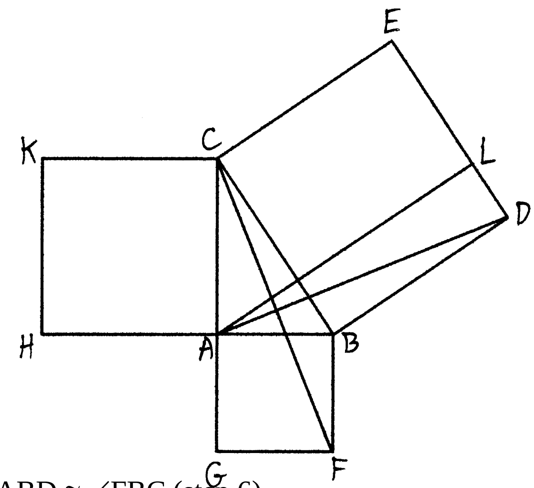
  How can we be sure that this is always true?

Individual Work

- **Create your own main lesson book pages**.
  Here are some ideas:
  - Write summaries of any of the proofs from *The Elements* we did in the past week, such as:
    I-13, 1-32, 1-47 (Pythag. Th.)
  - Create a page on “Equal Area Transformations”, including drawings with explanations.
  - Write an essay on constructible n-gons, including the idea that there are only 37 n-gons that are constructible with fewer than 300 sides.
  - Write an essay about your experience with this main lesson, perhaps focusing on how it challenged and stretched your thinking.
- Take the test found at the end of this document. For this test, you may use the two documents: “Summary of Book I”, and “Foundation of Euclid's Elements”.

Euclid's Proof of the Pythagorean Theorem

(Theorem I-47)



1. Given: ΔABC is a right triangle, with $\angle BAC$ a right angle.
2. Construct a square on each of the 3 sides of ΔABC.
3. Draw AL parallel to BD.
4. Draw lines AD and FC.
5. (a) $\angle DBC$ & $\angle FBA$ are both right angles.
   (b) $\angle DBC \cong \angle FBA$
   (c) $\angle DBC + \angle ABC = \angle FBA + \angle ABC$
   (d) $\angle ABD \cong \angle FBC$
6. (a) $BD \cong BC$ and $AB \cong FB$.
   (b) $\Delta ABD \cong \Delta FBC$ because $BD \cong BC$ and $AB \cong FB$ and $\angle ABD \cong \angle FBC$ (step 6).
7. (a) $\angle BAG$ is a right angle.
   (b) $\angle BAC$ and $\angle BAG$ are adjacent and both right angles, so CA is in a straight line with AG.
   (c) $\angle BAC \cong \angle FBA$
   (d) CG is parallel to FB.
   (e) [The area of] square GB is twice [the area of] ΔFBC, because they have the same base FB and lie between the same parallels FB and GC.
8. [The area of] parallelogram BL is twice [the area of] ΔABD, because they have the same base BD and they lie between the same parallels BD and AL.
9. $\Delta FBC \cong \Delta ABD$, therefore twice [the area of] ΔFBC is equal to twice [the area of] ΔABD.
10. [The area of] square GB is equal to [the area of] parallelogram BL.
11. Similarly, if lines AE and BK are drawn, parallelogram CL can be proven equal to square HC.
12. The sum of [the areas of] squares HC and GB is equal to
    the sum of [the areas of] parallelograms CL and BL.
13. [The area of] the square BE is equal to the sum of [the areas of] parallelograms CL and BL.
14. $\therefore$ **[The area of] the square BE is equal to the sum of [the areas of] the squares GB and HC.**

Q.E.D.

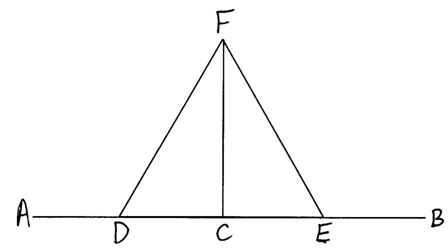
Test for Greek Geometry and Deductive Proofs

Part A

(1) Below is Euclid's proof of Theorem I-11. Fill in the reasons for each step.
(2 points per step)

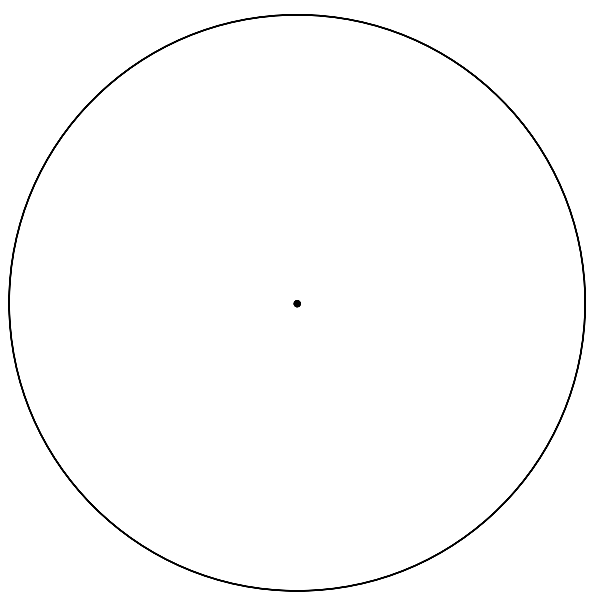
Theorem I-11

Construction of a line perpendicular to a given line (AB) from a point (C) on that line.



1. Given line AB and a point, C, on that line. Also, let point D be another random point on line AB.
2. Find point E on AB such that DC equals CE.
3. Construct an equilateral triangle DEF onto DE.
4. Draw line FC.
5. DF ≅ EF
6. ∠DCF ≅ ∠ECF, because the 3 sides of ΔDCF are congruent to the 3 sides of ΔECF.
7. CF is perpendicular to AB, since CF is standing on AB and ∠DCF and ∠ECF are both adjacent and equal. *Q.E.D.*

(2) Using a compass and straight edge, and only Euclidean-approved methods, construct a regular octagon (in other words, a "stop sign" shape) inside the given circle. (Show all construction lines.) (8 points)



(3) Give a summary of Euclid's proof of the Pythagorean Theorem (I-47) (8 points)

Part B - Short Answers

Write just a couple of sentences on each. (4 points each)

1. Would an indirect proof be acceptable to Euclid? (State why or why not.)

2. Why was the ratio of the diagonal to the side of a square so important to the Pythagoreans?

3. Why did Euclid's 5^{th} postulate get extra attention from later mathematicians?

4. What is an irrational number?

5. What is the difference between a postulate, a common notion, and a theorem?

6. What is an axiomatic system? What are its key components?

7. In Euclid's book, *The Elements*, what can be used to justify each of the steps in the proof of his very first theorem?