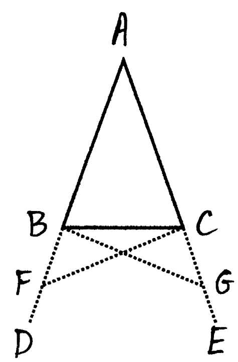
10^{th} Grade Assignment – Week #6

Group Assignment: *for Tuesday and Thursday* – work on the below problems, as you wish.

- **The Isosceles Triangle Theorem** states that with any isosceles triangles (which has two equal sides) the two base angles must also be equal. Euclid's proof of this theorem is the 5^{th} theorem of Book I. Keep in mind that with Euclid's proofs, the diagram he gives shows the final drawing. Therefore, you must imagine that the below drawing starts with only the triangle ABC, and then with each step the drawing may be added to.
  Your task is to (1) Read through the proof and make sure everyone understands it; (2) Justify each of the steps (answers are at the end of this document – but don't look until you've really worked on it!); (3) Write a brief summary of the proof.

  Theorem I-5 *In an Isosceles triangle, the base angles are equal to one another.*
  Proof: (the goal is to prove that $\angle ABC \cong \angle ACB$)



  1. Given isosceles ΔABC.
  2. Let AB and AC be the equal sides.
  3. Extend sides AB and AC to D and E, respectively.
  4. Choose point F at random on BD. Cut off AE at G, such that $AG \cong AF$.
  5. Join the lines FC and GB.
  6. Now look at ΔAFC and ΔAGB :
     they share $\angle GAF$; $AF \cong AG$ (step 4); $AB \cong AC$ (step 2)
     Therefore **$\Delta AFC \cong \Delta AGB$**; and then it follows that $FC \cong BG$; $\angle ACF \cong \angle ABG$; $\angle AFC \cong \angle AGB$
  7. Since $AF \cong AG$ and $AB \cong AC$ then $BF \cong CG$
  8. Now look at ΔBFC and ΔCGB :
     $\angle BFC \cong \angle CGB$ (same as $\angle AFC \cong \angle AGB$, step 6) $BF \cong CG$ (step 7) and $FC \cong BG$ (step 6)
     Therefore **$\Delta BFC \cong \Delta CGB$**, and then it follows that $\angle BCF \cong \angle CBG$, and $\angle CBF \cong \angle BCG$
  9. Since $\angle ACF \cong \angle ABG$ (step 6) and as part of these angles $\angle BCF \cong \angle CBG$ (step 8)
     Then the remaining angles are equal $\therefore \angle \mathbf{ABC} \cong \angle \mathbf{ACB}$ Q.E.D.

- **Which Polygons are Possible?**
  I already have mentioned to you that in 1796, at the age of 19, Carl Friedrich Gauss proved that the regular 17-gon is *constructible* (meaning that, with just a compass and straightedge we can place 17 points on a circle that are equally far apart). In fact, he also proved that all regular n-gons where $n = 2^{(2^n)}+1$ are also constructible. By letting n = 0, 1, 2, 3, 4 we get polygons with 3, 5, 17, 257, and 65537 sides respectively. Gauss also showed that there are 37 possible regular polygons under 300 sides that are constructible. For example, we have already seen that polygons with this many sides are constructible: 3, 4, 6, 12, 24, 48, 5, 10, 20, 15, 17. Your task is to complete this list of 37 constructible polygons. (Hint: What did I do to construct the 15-gon?)

(See next page →)

- **Equal Area Transformations**. I mentioned this at the end of last week's (extra) lecture #3. Your task is to find a method (using a compass and straightedge) to transform the first shape into the second shape such that the area remains the same.
    - Transform any triangle into a right triangle.
    - Transform any pentagon into a quadrilateral.
    - Transform any quadrilateral into a parallelogram.
    - Transform any parallelogram into rectangle.
    - Transform any rectangle into a square.

Individual Work

- **Create your own main lesson book pages**.
  Here are some ideas:
    - "Euclid's *Elements* and the axiomatic system".
      In a well written essay, summarize the keys point I have covered in my lectures, including key components of *The Elements*, and an explanation of what an axiomatic system is.
    - "Proof Summaries from *The Elements*"
      I have now presented you with several proofs from *The Elements*. Select a few of your favorite proofs, and write a brief summary of how he did it. Of course, this requires that you first read through the proof and understand exactly what he did. Your proof summary should not be simply copying the steps of what I did, but rather, leaving out the finer details and focusing on the key steps and ideas.
    - Write about anything that you have worked on in your work groups.

Announcement: For next week's group work, you will need to have a protractor.