9^{th} Grade Assignment – Week #26

Group Assignment:

For Tuesday

- **Al-Khwarizmi's Formula.**
  Do problem #1 from **Problem Set #6** (*Quadratic Formula* unit). The instructions are asking you to come up with a formula for the "root" (x). Note that with the problem he gives ($x^2 + 10x = 39$), the value for "b" is 10, and the value for "c" is 39. Your task is to write his instructions (in section IV) algebraically, so you end up with a formula for x in terms of b and c.
- Work together on the "homework" problems #2-9, from **Problem Set #6**.

For Thursday Math Magic Trick!

- Use algebra to figure out how the teacher can do the below magic trick.

  The teacher says: "Think of any three single-digit numbers (except zero), in any order, and write them down. Once you have chosen your three numbers, multiply the first number by 2, then add 5, and then multiply by 5. Now add the second number, subtract 4, multiply by 10, add 3, and then add the third number. Now tell me your final result and I will tell you what your original three numbers were."

Individual Work

- Finish any of the unfinished problems from Tuesday's group work (above).
- Solve these problems using the method of completing the square:
  (1) $x^2 + 4x + 3 = 0$
  (2) $x^2 – 7x – 30 = 0$
  (3) $6x^2 – 19x + 15 = 0$
- If you wish, write a "summary" or "main lesson book" page on al-Khwarizmi's work that we have

Problem Set #5

Classroom Discussion

Around 825AD, Mohammed ib'n Musa Al-Khwarizmi wrote *Hisab al-jabr wal-muqabala*. Most of the book focuses on arithmetic, measurement, business math and inheritance problems. But it is the first chapter, titled *On Calculating by Completion and Reduction*, that makes the book famous as the beginning of the formal study of algebra. We will now begin reading that first chapter.

On Calculating by Completion and Reduction

INTRODUCTION

When I considered what people generally want in calculating, I found that it is always a number. I also observed that every number is composed of units, and that any number may be divided into units.

Furthermore, I observed that the numbers which are required when calculating by completion and reduction are of three kinds, namely: roots, squares, and simple numbers[1]. Of these, a root is any quantity which is to be multiplied by a number greater than unity, or by a fraction less than unity. A square is that which results from the multiplication of the root by itself. A simple number [henceforth called only "number"] is any number which may be produced without any reference to a root or a square.

Of these three forms, then, two may be equal to each other, for example: squares equal to roots, squares equal to numbers, and roots equal to numbers[2].

Section I. CONCERNING SQUARES EQUAL TO ROOTS

The following is an example of squares equal to roots: "A square is equal to five roots". The root of the square then is five, and twenty-five forms its square, which is indeed equal to five times its root.

Another example: "One-third of a square equals four roots." Then the whole square is equal to 12 roots. So the square is 144, and its root is 12. Another such example: "Five squares equal ten roots." Therefore one square equals two roots. So the root of the square is two, and four represents the square.

In this manner, that which involves more than one square, or is less than one square, is reduced to one square. Likewise, the same is done with the roots; that is to say, they are reduced in the same proportion as the squares.

Section II. CONCERNING SQUARES EQUAL TO NUMBERS

The following is an example of squares equal to numbers: "A square is equal to nine." Then nine is the square and three is the root. Another example: "Five squares equal 80." Therefore one square is equal to one-fifth of 80, which is 16. Or, to take another example: "Half of a square equals 18." Then the whole square equals 36, and its root is six.

Thus any multiple of a square can be reduced to one square. If there is only a fractional part of a square, you multiply it in order to create a whole square. Whatever you do, you must do the same with the number.

Section III. CONCERNING ROOTS EQUAL TO NUMBERS

The following is an example of roots equal to numbers: "A root is equal to three." Then the root is three and the square is nine. Another example: "Four roots equal 20." Therefore one root is five, and the square is 25. Still another example: "Half a root is equal to ten." Then the whole root is 20 and the square is 400.

[In addition to the three above cases] I have found that these same three elements can produce three compound cases, which are:

Squares and roots equal to numbers,
Squares and numbers equal to roots, and
Roots and numbers equal to squares.

[These three cases are variations of a quadratic equation. Sections IV, V, and VI give methods for solving each of these cases. Only section IV, which deals with the first case, is covered in this workbook.]

[1] The term "roots" stands for multiples of the unknown, our x; the term "squares" stands for multiples of our x^2; "numbers" are constants.

[2] In our modern notation, this is $x^2 = bx$, $x^2 = c$, $x = c$.

Problem Set #6

Classroom Discussion

In section IV Al-Khwarizmi solves $x^2 + 10x = 39$.

Section IV. CONCERNING SQUARES AND ROOTS EQUAL TO NUMBERS

The following is an example of squares and roots equal to numbers: “A square and ten roots are equal to 39.” The question here is: “What must the square be such that when it is combined with ten of its own roots, it will amount to a total of 39?” To solve this, you take half the number of roots, which in this case gives us five. Then you multiply this by itself to get 25, and then add that result to 39, which gives us 64. Now take the root of this, which is eight, and subtract from it half the number of roots, resulting in three. This is the root of the square which you sought; the square itself is then nine.

This method is the same when you are given a number of squares. You simply reduce them to a single square, and in the same proportion you reduce the roots and simple numbers that are connected with them.

For example: “Two squares and ten roots equal 48.” The question therefore is: “What must the amount of the two squares be such that when they are summed up and then combined with ten times their root, the result will be a total of 48?” First of all it is necessary that the two squares be reduced to one. So we take half of everything mentioned in the statement. It is the same now as if the original question had been: “A square and five roots equal 24”, which means: “What must the amount of a square be such that when it is combined with five times its root, the result will be a total of 24?” To solve this, we halve the number of roots, which gives us 2½, and multiply that by itself, giving 6¼. To this we add 24, which yields a sum of 30¼, and then take the root of this, which is 5½. Subtracting half the number of roots, which is 2½, from this makes a remainder of three. This is the root of the square, and the square itself is nine.

Group Work

Al-Khwarizmi's Formula

1) Essentially, Al-Khwarizmi has just given a formula for solving quadratic equations. Consider the number of roots (i.e. the coefficient of the x term) to be b, and the constant to be c. Now reread Section IV in order to derive a formula for x given in terms of b and c.

Homework

Solve by completing the square.

2) $x^2 + 12x - 28 = 0$

3) $x^2 - 2x - 7 = 0$

4) $x^2 - 2x + 7 = 0$

5) $x^2 + 5x - 50 = 0$

6) $x^2 - 9x + 5 = 0$

7) $2x^2 + 11x + 12 = 0$

Solve for x in terms of the other variables or constants.

8) $ax - b^2 = d$

9) $a(x + b) = c$