9^{th} Grade Assignment – Week #25

Announcement:

- Please note that in this unit, we will not do all of the problem sets from the workbook, because much of the material found in the workbook problem sets are actually covered in the lectures. Therefore, we will skip some of the problem sets.

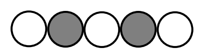
Group Assignment:

For Tuesday

1) *Solving a Quadratic Equation by Completing the Square*.
   Read through the example on **Problem Set #3** (Quadratic Formula unit), and make sure you understand it. This is the same thing I did in Monday's lecture.
2) As I did in Monday's lecture, solve the following quadratic equation in two different ways: first using factoring, and then using the method of completing the square.
   $x^2 + 10x = -21$
3) Solve the following quadratic equation using the method of completing the square:
   $x^2 + 6x = 3$
4) (If you still have time) Do problems #1-5 on **Problem Set #3** (Quadratic Formula unit).

For Thursday (We started this puzzle last week.)

5) There are five coins – two of one type, and three of another type. These coins are in a line as shown here.



The objective is to move them in such a way that they end up still in a contiguous line of five, but that the two black coins are side-by-side, and the three white coins are also side-by-side. The rule is that you must move two coins at a time, and those two coins must be touching each other the entire time that you are moving them. (No, you can't push them together to get rid of any gaps.)

a) What is the fewest number of moves in which this is possible? (This is the one from last week's assignment, and the one I did in Monday's lecture. Be sure you have it correct before proceeding to the below problems.)

b) How can you do this in three moves if the two coins you are moving cannot switch their left-right orientation?

c) How can you do this in three moves if the two coins being moved must be of different colors?

d) How can you do this in four moves if the two coins being moved must be of different colors, and switching isn't allowed?

Individual Work

- Do problems #6-13 on **Problem Set #3** (Quadratic Formula unit).

Problem Set #3

Group Work

Greek Geometric Puzzles

1) A rectangle has a length of 6 inches and a height equal to the length of the side of a square. Find the side of the square such that the square has an area that is 55 square inches greater than the rectangle.

2) A rectangle has a length of 8 inches and a height equal to the length of the side of a square. Find the side of the square such that the rectangle has an area that is 12 square inches greater than the square.

3) A rectangle has a length of 6 inches and a height equal to the side of a square. Find the side of the square such that the sum of the areas of the two figures is 20 square inches.

Solving a Quadratic Equation
by Completing the Square

Study the following example and make sure that you understand it well.

Example: $x^2 + 10x - 24 = 0$

$x^2 + 10x = 24$

$x^2 + 10x + 25 = 24 + 25$

$(x + 5)^2 = 49$

$\sqrt{(x+5)^2} = \sqrt{49}$

$|x+5| = 7$

$x+5 = \pm 7$

$x = -5 \pm 7$

$x = -12, 2$

4) It would have been easier to solve the above example by factoring. Why do you think this new method is important?

Solve by Completing the Square (as shown above).

5) $x^2 + 8x + 12 = 0$

Homework

Solve by getting the squared term alone and then square rooting both sides, as was done on the previous set.

6) $(x - 2)^2 = 100$

7) $(x + 9)^2 = 1$

8) $(x + 5)^2 = 7$

9) $(x - 3)^2 = -4$

10) $\left(x + \frac{3}{8}\right)^2 = \frac{9}{4}$

11) $\left(x - \frac{1}{3}\right)^2 = \frac{5}{9}$

Solve by completing the square.

12) $x^2 - 6x + 5 = 0$

13) $x^2 + 4x - 21 = 0$

Problem Set #5

Note: We will read part of this in this week's second lecture.

Around 825AD, Mohammed ib'n Musa Al-Khwarizmi wrote *Hisab al-jabr wal-muqabala*. Most of the book focuses on arithmetic, measurement, business math and inheritance problems. But it is the first chapter, titled *On Calculating by Completion and Reduction*, that makes the book famous as the beginning of the formal study of algebra. We will now begin reading that first chapter.

On Calculating by Completion and Reduction

INTRODUCTION

When I considered what people generally want in calculating, I found that it is always a number. I also observed that every number is composed of units, and that any number may be divided into units.

Furthermore, I observed that the numbers which are required when calculating by completion and reduction are of three kinds, namely: roots, squares, and simple numbers[1]. Of these, a root is any quantity which is to be multiplied by a number greater than unity, or by a fraction less than unity. A square is that which results from the multiplication of the root by itself. A simple number [henceforth called only "number"] is any number which may be produced without any reference to a root or a square.

Of these three forms, then, two may be equal to each other, for example: squares equal to roots, squares equal to numbers, and roots equal to numbers[2].

Section I. CONCERNING SQUARES EQUAL TO ROOTS

The following is an example of squares equal to roots: "A square is equal to five roots". The root of the square then is five, and twenty-five forms its square, which is indeed equal to five times its root.

Another example: "One-third of a square equals four roots." Then the whole square is equal to 12 roots. So the square is 144, and its root is 12. Another such example: "Five squares equal ten roots." Therefore one square equals two roots. So the root of the square is two, and four represents the square.

In this manner, that which involves more than one square, or is less than one square, is reduced to one square. Likewise, the same is done with the roots; that is to say, they are reduced in the same proportion as the squares.

Section II. CONCERNING SQUARES EQUAL TO NUMBERS

The following is an example of squares equal to numbers: "A square is equal to nine." Then nine is the square and three is the root. Another example: "Five squares equal 80." Therefore one square is equal to one-fifth of 80, which is 16. Or, to take another example: "Half of a square equals 18." Then the whole square equals 36, and its root is six.

Thus any multiple of a square can be reduced to one square. If there is only a fractional part of a square, you multiply it in order to create a whole square. Whatever you do, you must do the same with the number.

Section III. CONCERNING ROOTS EQUAL TO NUMBERS

The following is an example of roots equal to numbers: "A root is equal to three." Then the root is three and the square is nine. Another example: "Four roots equal 20." Therefore one root is five, and the square is 25. Still another example: "Half a root is equal to ten." Then the whole root is 20 and the square is 400.

[In addition to the three above cases] I have found that these same three elements can produce three compound cases, which are:

Squares and roots equal to numbers,
Squares and numbers equal to roots, and
Roots and numbers equal to squares.

[These three cases are variations of a quadratic equation. Sections IV, V, and VI give methods for solving each of these cases. Only section IV, which deals with the first case, is covered in this workbook.]

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[1] The term "roots" stands for multiples of the unknown, our x; the term "squares" stands for multiples of our x^2; "numbers" are constants.

[2] In our modern notation, this is $x^2 = bx$, $x^2 = c$, $x = c$.