Tutorial Session Notes
Grade 7
Quarter #4 (Week 25-32)

<u>About these notes</u>:

- These notes are primarily for those who are acting as the tutor – either a parent or a class teacher.
- In the first year of JYMA, Maria (our JYMA tutor) and I met every week and talked about grades 5-8, and we made a list of suggested topics for the Friday tutorial session.
- In order to support those who are acting as the tutor for their child or a whole class, I am sharing these notes with those who are acting as the tutor.
- Of course, these tutorial sessions are also an opportunity for the students to ask their tutor questions.
- If you are acting as the tutor, it may be helpful to read the section of the JYMA Handbook titled "The Role of the Tutor".

Week #25

- Ask them if they feel they made improvements with their math tricks.
- Play NIM – and see if they can develop the "unbeatable" strategy.
    - <u>Example</u>: Play one-pile NIM starting with 7 gems, where the rule is that with each turn you can remove 1, 2, or 3 gems. Do you want to go first? What is the best move? Once they know how to win starting with this many gems, slowly build up the number of gems you start with.
    - See below notes (from last week).
- Ratio Practice
    - If you have a rectangle and it measures 71cm on the base and the height is 40cm, what is the ratio of base to height *in decimal form*? (You need to do long division.) Answer B:H = 1.775 : 1
    - If you drive one day for three hours and you go 200 miles, and the next day you drive four times as long, how far will you go? (This is a case of "directly proportional".)
    - If one day you drive to your uncle's house and it takes 27 minutes, and the next day you drive ¾ as fast as the first day, how long is it going to take you then? (Answer: $^4/_3$ as long = 36 min)
    - Do a seesaw problem: David weighs 60kg and is 3meters from the fulcrum. If Jim is balancing the seesaw by sitting 2m from the fulcrum, how much does he weigh?
      Weight and Distance are Inversely Proportional
      Ratio of Distances $\rightarrow$ D:J = 3:2
      Ratio of Weights must therefore be $\rightarrow$ D:J = 2:3
      Jim's weight $\rightarrow$ $J = {}^3/_2 \cdot D \rightarrow J = {}^3/_2 \cdot 60 \rightarrow$ **<u>J = 90</u>**
- Notes for NIM (taken from last week's Tutorial Notes).
    - The goal for the students is to figure out the unbeatable strategy.
    - It may be best for the students to practice against you, where you play the role of the flawless "NIM Machine". When a student, or group of students is fully confident, they can challenge you. You get to choose the number of gems to begin the game. The student(s) then choose whether they would like to go first, or have the teacher go first. If the students execute the unbeatable strategy without any mistake, then they should win every time. But if they make a mistake, they will lose – presuming that you don't make a mistake!
    - <u>Note to tutor</u>: You have to learn the perfect strategy before doing this.

  - Here are the unbeatable strategies for the four variations:
- The unbeatable strategy for One Pile NIM "*winner* takes the last gem" (and each turn you can remove 1, 2, or 3 gems). Fairly quickly, students realize that if they leave their opponent with 4 gems, then they will win. They will also win if they leave their opponent with 8, 12, or 16 gems – any multiple of four – assuming, of course, that no mistakes are made. Lastly, we need to know who should go first. If given the choice, a player should have their opponent go first if the beginning number of gems is a multiple of 4. Otherwise (if the beginning number of gems is not a multiple of 4), the player should elect to go first and then leave the opponent with a multiple of 4.
- The unbeatable strategy for One Pile NIM "*loser* takes the last gem".
  The strategy is nearly the same as above. However, in this case, we realize that we will win if we leave our opponent with 1, 5, 9, 13, etc., gems. In other words, our goal is to leave our opponent with one more than a multiple of 4.
- Note: At the highest skill level, with each version of One-Pile NIM (shown above), we can change the rule (before we start playing) so that with each turn you can remove up to 4 gems (or a different number) – and still the student should be able to win.
- The unbeatable strategy for both versions of Two-Pile NIM. Surprisingly, whether the rule states that the last player to remove a gem wins or loses, the strategy is essentially the same. Either way, if we can leave our opponent with two gems in each pile, we will win. If we begin with an equal number in each pile, then we want our opponent to go first. Otherwise, we will go first and leave our opponent with an equal number in each pile until it gets down to two gems in each pile.

Week #26

- Important: Ask them what their results were for the ratios of a circle question (Sheet #5, problem #7)
  - Ask them to share with each other what their measurements were, and what the results were when they divided.
- Play 2 pile nim - start with 3 and 7, 5 and 7
- Yes, I'm still wanting them to get better at NIM.
  - In Wednesday's lecture I showed them how to build up to a strategy for 1-pile NIM where taking the last gem wins. Use a similar approach to develop the "unbeatable" strategy for the other variations. See below notes (from previous weeks).
  - The unbeatable strategy for One Pile NIM "*winner* takes the last gem" (and each turn you can remove 1, 2, or 3 gems). Fairly quickly, students realize that if they leave their opponent with 4 gems, then they will win. They will also win if they leave their opponent with 8, 12, or 16 gems – any multiple of four – assuming, of course, that no mistakes are made. Lastly, we need to know who should go first. If given the choice, a player should have their opponent go first if the beginning number of gems is a multiple of 4. Otherwise (if the beginning number of gems is not a multiple of 4), the player should elect to go first and then leave the opponent with a multiple of 4.
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- If time, go over square root algorithm sheet #1 page 92.

Week #27

- Rate problems: (Hopefully this is fairly quick.)
  - If somebody is biking at 18 mph, how far did they go in 3 hours?
  - If somebody is biking at 18 mph, how far did they go in 2 hours and 20 minutes?
  - If somebody is biking at 18 mph, how long does it take to go 36 miles?
  - If somebody is biking at 18 mph, how long does it take to go 84 miles?
  - What is your speed if you take 4 hours to hike 12 miles?
  - What is your speed if it takes 6 hours to go 200 miles?
- Geometry: (Hopefully this is fairly quick.)
  - Go over the geometry problems on p. 80
  - Make up similar problems to make sure they understand.
- Square roots:
  - $\sqrt{418609}$ Ask: how many digits are in the answer, and what is the first digit?
  - $\sqrt{5669161}$ Ask: how many digits are in the answer, and what is the first digit?
- The Squaring Formula: (See page 95 in workbook for explanation.)
  - $(a+b)^2 = a^2 + b(2a+b)$
  - Show how to use the squaring formula to calculate. Do these:
    - 24^2
    - 58^2
- Four Ratios in a Circle
  - Ask them what are the 4 ratios of a circle:
    - C:D ≈ 3.14:1
    - D:C ≈ 0.318:1
    - C:D ≈ 22:7
    - D:C ≈ 7:22
  - What does each one mean?
  - 2 simple problems to use to solve it:
    - What is the circumference of a circle if the diameter is 5 meters?
      - Can solve in 3 different ways:
        5 x 3.14 ≈ 15.7
        $\frac{22}{7}$ x 5 ≈ $\frac{110}{7} \rightarrow 15\frac{5}{7}$
        (not recommended!) 5÷0.318 ≈ 15.72
    - What is the diameter of a circle if the circumference is 33 meters?
      - Can solve in 3 different ways
        33 x 0.318 ≈ 10.494
        $\frac{7}{22}$ x 33 ≈ 10½
        (not recommended!) 33÷3.14

Week #28

- Ask if any questions with **Rates Sheet #2** (p70-71).
- Give these rate problems:
    - How far do you go in 5 hours and 15 minutes at 24 mph?
    - How long does it take to go 50 miles at 20 mph?
    - What is your speed if you go 210 miles in 7 hours?
    - What is your fuel efficiency if you go 210 miles with 7 gallons of gas?
- Ask if any questions with **Geometry Sheet #2**.
    - Make sure especially that they understand #8 and #9
    - Ask them to explain #10.
- **Square Root Algorithm**
    - Ask if they were able to do the square root algorithm problems #3a and 3b p. 96.
    - Do problems 3c and 3d (This is quite challenging!)

Week #29

- Go over **Geometry Sheet #3** (p84-5). Especially make sure they understand #3 and #5
- Go over **Rates Sheet #3** (p72-3). Especially make sure they understand #8, #9, and #13.
- **Square Root Algorithm**
    - $\sqrt{7203856}$ How many digits does the answer have, and what is the first digit? (We will figure out the answer to this next week in the lecture.)
    - $\sqrt{720801}$ How many digits will the answer have, and what is the first digit?
    - Use the "Long Algebraic Method" (which is what I've been doing in the lecture) to find:
        1) $\sqrt{3969}$
        2) $\sqrt{7396}$
- If time, do a couple Pythagorean Theorem problems, such as:
    - Legs/Sides are 9m and 12m, find the hypotenuse.
    - Legs/Sides are 7m and 9m, find the hypotenuse.

Week #30

- **Rates** practice
    - If someone bikes 12 miles uphill at 6mph how long does it take them?
    - And then they come down at 24 mph, how long does it take to come down?
    - What is the average speed for the whole trip? (avg speed = total distance / total time) (Answer: 24 ÷ 2.5 = 9.6 mph)
- Go over problems from rate sheet #4, in particular make sure they understand #11-13.
- **Square Root Algorithm** problems. Use the method explained in the lectures, which is also given on **Sheet #4** (p97)
    - $\sqrt{218089}$
    - $\sqrt{55175184}$
- If extra time, do **Rates Sheet #4** #7 (p. 74)

Week #31

- Make sure they understand **Rates Sheet #6** (p78) specifically #3-7
- **Geometry Sheet #5** (p88): Go over all three problems with Problem #2.
  Ask them how many degrees are in a triangle? Ask how many degrees are in a quadrilateral?
- **Square Root Algorithm**
  - Together, do one or both of these problems
    by using the method of "Short Algebraic Method" (as shown on p99)
    1) $\sqrt{466489}$
    2) $\sqrt{17123044}$
    (strongly recommend that the tutor works it out before the tutorial session)

Week #32

- Do something fun! Bring the year to a nice close!