

A Basic Introduction to Functions

Problem Set #1

Function Notation

At this point, we are comfortable with seeing a mathematical relation expressed between x and y , such as:
 $y = x^2 + 3x - 8$

We can then ask a question like, “*What is the value of y when x equals 4?*”, and then calculate that it is
 $y = 4^2 + 3(4) - 8 = 20$

In higher level mathematics, this same relation is often expressed in *function notation* as:

$$f(x) = x^2 + 3x - 8$$

“ $f(x)$ ” should be read as “*f of x*”.

And the same question can then be expressed simply as “*Find $f(4)$* ”.

As with learning any new language, the language of functions can at first seem rather strange, but after a short while it will seem like second nature.

Evaluate each function:

Example: Find $f(3)$ given that
 $f(x) = 5x - 7$

Solution: We simply put 3 into x on the right side of the equals sign, and get
 $5 \cdot 3 - 7$.

Therefore $f(3) = 8$.

- 1) Find $f(5)$ given that
 $f(x) = x^2 - 6$.
- 2) Find $f(2)$ given that
 $f(x) = 3x^2 + 4x + 1$.
- 3) Find $f(16)$ given that
 $f(x) = 2x + \sqrt{x}$
- 4) Find $g(-3)$ given that
 $g(x) = 4x + 8$
- 5) Find $h(0)$ given that
 $h(y) = y^2 - 7y + 3$

Problem Set #2

Given these function definitions:

$$f(x) = 3x^2 - 4x + 8$$

$$g(x) = 5x - 3$$

$$h(x) = x^2$$

Find each value.

- 1) (skip #1)
- 2) $f(10)$
- 3) $g(10)$
- 4) $h(10)$
- 5) $f(-3)$
- 6) $g(1/2)$
- 7) $h(-7)$
- 8) $f(0)$
- 9) $g(0)$
- 10) $h(1/3)$
- 11) $f(\sqrt{5})$
- 12) $g(\sqrt{5})$
- 13) $h(\sqrt{17})$
- 14) $h(\sqrt{5})$

Problem Set #3

Given these function definitions:

$$f(x) = 3x^2$$

$$g(x) = 2x - 7$$

Find each value.

- 1) $f(10)$
- 2) $g(10)$
- 3) $f(g(10))$
- 4) $g(f(10))$
- 5) $f(g(x))$
- 6) $g(f(x))$
- 7) $g(f(x + 3))$

Problem Set #4

- 1) Find each of the following given that:

$$f(x) = x^2 + 5x + 3$$

$$g(x) = \sqrt{x-10}$$

$$h(x) = 4x - 2$$

- a) $f(6)$ h) $h(f(-3))$
b) $g(14)$ i) $f(g(x))$
c) $g(20)$ j) $f(h(x))$
d) (skip) k) $g(h(x))$
e) $h(x^3)$ l) $h(f(x))$
f) $f(3x)$ m) $f(h(g(11)))$
g) $f(h(0))$

- 2) Find each of the following given that:

$$f(x) = x^2 - 4$$

$$g(x) = \sqrt{x+5}$$

$$h(x) = 3x + 5$$

- a) $f(5)$ g) $f(f(3))$
b) $g(20)$ h) $f(g(7))$
c) $h(\frac{2}{3})$ i) $f(h(-2))$
d) $f(-3)$ j) $f(h(x))$
e) $g(-3)$ k) $h(f(x))$
f) $g(x^2-7)$ l) $h(f(g(x)))$

Problem Set #5

Domain and Range

In higher level mathematics, it is important to have a good understanding of the terminology and language used with functions. In the beginning such language may seem awkward. We will now introduce the idea of *domain* and *range*.

Domain is the set of real numbers that can be put into a given function, and *range* is the set of real numbers that can possibly come out of the function.

- 1) For each of the following functions, explain why the domain and range is as stated.

a) $f(x) = x^2 - 4$

The domain is x can be any real number.

The range is $f(x) \geq -4$

b) $g(x) = \sqrt{x+5}$

The domain is $x \geq -5$

The range is $g(x) \geq 0$

c) $h(x) = 3x + 5$

The domain is x can be any real number.

The range is $f(x)$ can be any real number.

- 2) Give the domain and range of:

a) $f(x) = -3x^2 + 5$

b) $f(x) = 8 - \sqrt{x+3}$

- 3) Give the domain and range of:

a) $f(x) = \frac{5}{x}$

b) $f(x) = x^2 - 7$

c) $f(x) = \sqrt{x-5} + 10$

Answer Key

Problem Set #1

- 1) 19
- 2) 21
- 3) 36
- 4) -4
- 5) 3

Problem Set #2

- 1) (Skip #1)
- 2) 268
- 3) 47
- 4) 100
- 5) 47
- 6) $-\frac{1}{2}$
- 7) 49
- 8) 8
- 9) -3
- 10) $\frac{1}{9}$
- 11) $23 - 4\sqrt{5}$
- 12) $-3 + 5\sqrt{5}$
- 13) 17
- 14) 5

Problem Set #3

- 1) 300
- 2) 13
- 3) 507
- 4) 593
- 5) $3(2x - 7)^2 = 12x^2 - 84x + 147$
- 6) $6x^2 - 7$
- 7) $6(x + 3)^2 - 7 = 6x^2 + 36x + 47$

Problem Set #4

- 1)
 - a) 69
 - b) 2
 - c) $\sqrt{10}$
 - d) Skip
 - e) $4x^3 - 2$
 - f) $9x^2 + 15x + 3$
 - g) -3
 - h) -14
 - i) $x + 5\sqrt{x - 10} - 7$
 - j) $16x^2 + 4x - 3$
 - k) $2\sqrt{x - 3}$
 - l) $4x^2 + 20x + 10$
 - m) 17
- 2)
 - a) 21
 - b) 5
 - c) 7
 - d) 5
 - e) $\sqrt{2}$
 - f) $\sqrt{x^2 - 2}$
 - g) 21
 - h) 8
 - i) -3
 - j) $9x^2 + 30x + 21$
 - k) $3x^2 - 7$
 - l) $3x + 8$

Problem Set #5

- 1) Answers may vary
- 2)
 - a) Domain: \mathbb{R}
Range: $f(x) \leq 5$
 - b) Domain: $x \geq -3$
Range: $f(x) \leq 8$
- 3)
 - a) Domain: \mathbb{R} except $x \neq 0$.
Range: \mathbb{R} except $f(x) \neq 0$
 - b) Domain: \mathbb{R} .
Range: $f(x) \geq -7$
 - c) Domain: $x \geq 5$.
Range: $f(x) \geq 10$.