7th Grade Assignment – Week #31

Group Assignments:

For Tuesday:

- Do **Geometry – Sheet #4:** Problems #3 and #4.
- With however much time you still have in your group work session, work together on **Rates – Sheet #6:** Problems #3-7.

For Thursday:

- Do **Square Root Algorithm – Sheet #6** – see how many problems you can do from #2 (at the bottom of the page). It may help you to organize your work in a similar manner as shown in the example.

Individual Work

- Whatever problems you didn't complete in your group (see above), you may wish to complete on your own.
- Do **Geometry – Sheet #5**. Do as many problems as you can.

Rates – Sheet #6

1) A small plane flies 875 miles in 4 hours and 10 minutes.
   a) What is the average speed for the flight?
   b) At that rate of speed, how far does the plane fly in 5 hours and 45 minutes?
   c) At that rate of speed, how long does it take for the plane to fly 1575 miles?

2) A car uses 2½ gallons of gas in 70 miles.
   a) What is its fuel efficiency (in mpg) for the trip?
   b) Given the above fuel efficiency, how far can the car go on 19 gallons of gas?

3) Noah biked 72 miles in 4 hours yesterday, which is an average speed of 18 mph.
   a) If he bikes $^{8}/_{9}$ as fast today, then how long will it take him to cycle the 72 miles?
   b) If he bikes $^{9}/_{8}$ as fast today, then how long will it take him to cycle the 72 miles?
   c) If he bikes $^{8}/_{9}$ as fast today, then how far will he cycle in 4 hours?
   d) If he bikes $^{9}/_{8}$ as fast today, then how far will he cycle in 4 hours?

4) Water is leaking out of a tank at a rate of 20mℓ every minute. How long does it take 6ℓ to leak out of the tank?

5) Benny biked up a 3-mile long hill in 30 minutes, and then came back down the same route in 6 minutes. What was his average speed…
   a) Going up the hill?
   b) Going down the hill?
   c) Round trip?

6) Sue biked 6 miles to the beach at 20 mph and returned (along the same route) at a rate of 12 mph. What was her average speed for the whole trip?

7) On September 14, 2003, Tim Montgomery broke the world record by running the 100m dash in 9.87 seconds.

a) What was his average speed for the race in m/s?

b) *Challenge!* Calculate his average speed in km/h.

c) *Challenge!* Calculate his average speed in mph.
(Hint: 1km ≈ 0.62mi)

8) *Challenge!* A plane leaves Denver at 2:10 and flies at a rate of 600 mph toward New York, 1800 miles away. At 2:30, another plane leaves New York toward Denver at a rate of 500 mph. At what time, and how far from Denver, do they pass?

9) *Challenge!* A thief crosses a bridge at 9:37pm going 60 mph in a car. At 9:49pm, a police car chasing the thief and going 75 mph crosses the same bridge. Assuming that the cars maintain their speeds, at what time, and how far from the bridge, does the police car catch the thief?

Mental Math

10) Cross multiply.

$$\begin{array}{r} 26 \\ \times\ 33 \end{array}$$

11) $216 - 197 =$

12) $332 \div 5 =$

13) $155 \cdot 4 =$

14) $260 \cdot 11 =$

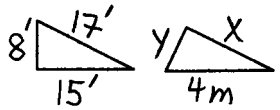
15) $63 \cdot 67 =$

16) $296.5 \div 100 =$

17) $35 \cdot 11 =$

Review

18) Find X and Y given that the two triangles are similar.

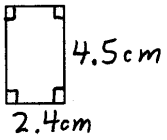


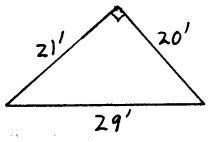
19) What is 4800 decreased by 62½%?

20) What is 42 increased by 350%?

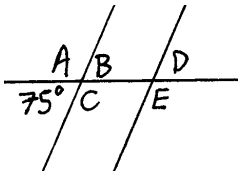
Geometry – Sheet #4

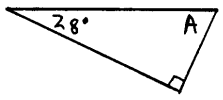
1) Find the area and perimeter.

a) 

b) 

2) Find each variable.

a) 

b) 

3) **Using Pythagoras's formula to find Pythagorean triples.**

A Pythagorean triple is a special right triangle where all three sides (X, Y, and Z) have lengths that are whole numbers. Pythagoras's formula is:

$$X = 2n + 1$$
$$Y = 2n^2 + 2n$$
$$Z = 2n^2 + 2n + 1$$

The table below shows different Pythagorean triples by choosing different values for n. Fill in the table by using the formulas given above.

n	X	Y	Z
1			
2			
3			
4			
5			
6			

4) **Using the Arabian formula to find Pythagorean triples.**

The Arabian formula is (where $u > v$):

$$X = u^2 - v^2$$

$$Y = 2uv$$

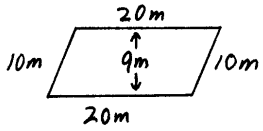
$$Z = u^2 + v^2$$

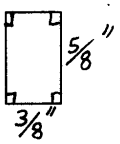
The table below shows different Pythagorean triples by choosing different values for u and v. Fill in the table by using the formulas given above.

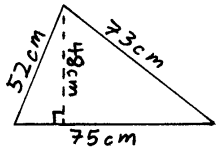
u	v	X	Y	Z
2	1			
4	1			
6	1			
8	1			
3	2			
5	2			
7	2			
9	2			
4	3			
8	3			
5	4			
7	4			
9	4			
6	5			
8	5			
7	6			

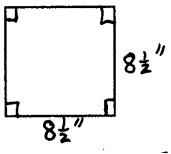
Geometry – Sheet #5

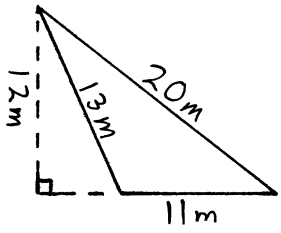
1) Find the area.

a) 

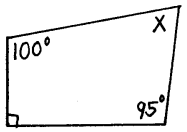
b) 

c) 

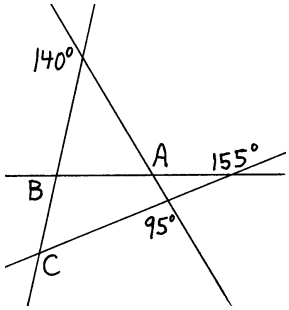
d) 

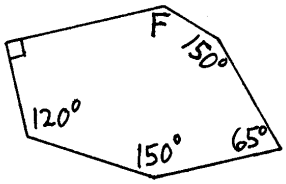
e) 

2) Find the variables.

a) 

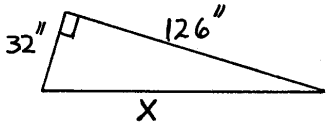
b) *Challenge!*

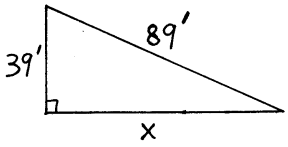


c) 

3) What is the measure of the interior angles of a regular hexagon?

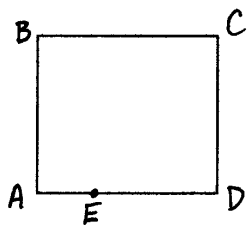
4) Calculate the value for X.

a) 

b) 

5) Given that ABCD is a rectangle, and that:

$\overline{AB}$ = 9.6m
$\overline{BC}$ = 11m
$\overline{AE}$ = 2.8m



a) What is the area of the rectangle ABCD?

b) What is the area of triangle AEB?

c) What is the area of triangle AEC?

d) What is the area of trapezoid CDEB?

e) What is the area of the triangle BCE?

f) What is the area of the triangle BDE?

6) Looking back at the previous worksheet, use the Arabian formula to calculate the values of X, Y, and Z for…

a) u=5 and v=3

b) u=6 and v=3

7) The Pythagorean triples that were listed in the Arabian formula table on the previous worksheet were special.

a) What was special about them?

b) Why didn't that table include your answers from the above problem (using u=5, v=3; or u=6, v=3)?

Mental Math

8) $21 \div 999 =$

9) $40 \cdot 69 =$

10) $700 \cdot 600 =$

11) $7012 - 5996 =$

12) $45^2 =$

13) $7 \cdot 99 =$

14) $34 \cdot 26 =$

15) $900 \div 150 =$

Review

16) At 48 mph, how far does a car travel in 2 hours and 45 minutes?

17) Pam leaves her house at 11:10am, to cycle to Karen's house, which is 39 miles away. At what time will she arrive if she averages a rate of 15 mph?

$\sqrt{\ }$ Algorithm − Sheet #6

1) Calculate each square root using the *Long Algebraic Method*, as done on the previous worksheets. (All answers work out exactly.)

a) $\sqrt{403225}$ b) $\sqrt{61009}$ c) $\sqrt{24137569}$

The Short Algebraic Method

The Basic Idea

- *Reducing the amount of Calculating*. The long algebraic method, described above, requires some tedious, and unnecessary, calculations, which can be eliminated.
- Look at example for the long algebraic method shown on sheet #4. Looking at the left side of each step, we see, for step #1: $n - a_1^2$, and then for step #2: $n - a_2^2$, etc.
- Since $a_2 = a_1 + b_1$, we can use the *Squaring Formula* $(a+b)^2 = a^2 + b(2a+b)$ to get:

$$a_2^2 = (a_1 + b_1)^2 = a_1^2 + b_1(2a_1 + b_1)$$

This is the key idea: In place of subtracting a_2^2 from n, we can instead subtract the whole of $\{a_1^2 + b_1(2a_1 + b_1)\}$ from n since it is equal to a_2^2. This seems like more work, but it's not - it's less work.

In other words, instead of doing $n - a_2^2$, we can do $n - \{a_1^2 + b_1(2a_1 + b_1)\}$,
which is the same as $(n - a_1^2) - \{b_1(2a_1 + b_1)\}$

In short: instead of doing $n - a_2^2$ we do $(n - a_1^2) - \{b_1(2a_1 + b_1)\}$

Likewise, instead of doing $n - a_3^2$ we do $(n - a_2^2) - \{b_2(2a_2 + b_2)\}$

Likewise, instead of doing $n - a_4^2$ we do $(n - a_3^2) - \{b_3(2a_3 + b_3)\}$, etc.

Of course, any sane person would ask, "Haven't we made things more complicated?". The answer to this is (and this is where the genius of this method comes in): $(n - a_2^2) - \{b_2(2a_2 + b_2)\}$ is easier to do than $n - a_3^2$ because a_3^2 requires us to square some big ugly number (e.g. 2680), whereas we have already calculated both $(n - a_2^2)$ (which is 443856 in the example below)
and $\{b_2(2a_2 + b_2)\}$ (which is 422400 in the example below).

Subtracting 443856 − 422400, is easier than squaring 2680!!!!!

Example: $\sqrt{7203856}$ (once again!):

	n	7203856	our first estimate (a_1) is 2000.	
	$\mathbf{a_1^2}$	− 4000000		
step #1	$\mathbf{n - a_1^2}$	3203856	$= b_1(4000 + b_1)$	$\rightarrow$ $b_1 = 600$
	$\mathbf{b_1(2a_1 + b_1)}$	− 2760000	←	
step #2	$\mathbf{n - a_2^2}$	443856	$= b_2(5200 + b_2)$	$\rightarrow$ $b_2 = 80$
	$\mathbf{b_2(2a_2 + b_2)}$	− 422400	←	
step #3	$\mathbf{n - a_3^2}$	21456	$= b_3(5360 + b_3)$	$\rightarrow$ $b_3 = 4$
	$\mathbf{b_3(2a_3 + b_3)}$	− 21456	←	
		0	So our answer is *exactly*	**2684**

2) Calculate each square root using the *Short Algebraic Method*. It is important that you do the problem and organize your work exactly like the example given above. Notice that the first three problems are the same ones given in the previous exercise. (All answers work out exactly.)

a) $\sqrt{403225}$ b) $\sqrt{61009}$ c) $\sqrt{24137569}$ d) $\sqrt{393129}$ e) $\sqrt{145924}$