

8th Grade Assignment – Week #22

Group Assignments:

For Tuesday:

- Look over all the problems on *Proportions & Dimensional Analysis Practice Sheet #1* and **Group Sheet #1**, and choose the more difficult ones to do in your group work. (The rest of the problems should be done individually.)

For Thursday: Puzzle!

- **The Grains of Rice Classic Puzzle**

The Story:

There once was a king in India whose son was killed in battle. A wise but poor man in his kingdom invented the game of chess to help the king with his grieving. The king enjoyed the game so much that he invited the wise man to his castle and told him that, as a reward, he could have anything in the kingdom that he desired. The wise man thought for a moment and then said that since his village sometimes did not have enough food, he would like a good amount of rice.

When the king asked him how much rice he would like, the wise man stated his answer as a puzzle. He said that a single grain of rice should be placed on the first square of the chessboard (which has a total of 64 squares). Then two grains of rice should be placed on the second square, and then double that amount (4 grains) on the third square, and double that amount (8 grains) on the fourth square, and so on up to the last square – square #64. That is how much rice he would like – if the king didn't feel that the request was too great. He warned the king that all the rice wouldn't fit nicely on the chessboard, but that didn't really matter – he just wanted that amount of rice. The king thought to himself that the wise man was actually quite a fool since he could have had anything in the kingdom, and he was only requesting a few bags of rice.

The Questions:

- a) How many grains of rice are there on the whole chessboard (assuming that it would somehow fit)?
- b) How many 25-pound sacks of rice would this be, and if all these sacks were laid in a line end-to-end, how far would they stretch? (Each sack is 20 inches long and contains around 400,000 grains of rice.)
- c) What is the volume of the rice? (Hint: There are about 400 grains of rice in a tablespoon.)

Individual Work

- Take the **Percents & Growth Test** found at the end of this document.
- From *Proportions & Dimensional Analysis Practice Sheet #1* and **Group Sheet #1**, do whatever problems you didn't finish during your group meeting you should finish individually.

Proportions & Dimensional Analysis – Practice Sheet #1

Note: Throughout this whole unit you should: (1) use the *Conversion Table* at the back; (2) use a calculator; (3) write down what goes into your calculator; (3) round your answers to three significant digits, when necessary.

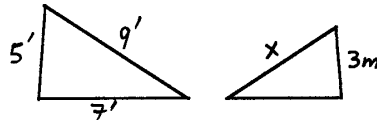
- 1) John earns \$52.40 in an 8-hour day. What is his hourly wage?
- 2) At an hourly wage of \$5.35 per hour, how much does a worker make...
 - a) in an 8-hour day?
 - b) in a 40-hour week?
 - c) in a 50-week year?
- 3) Gail biked 70 miles in four hours. What was her average speed?
- 4) How long does it take to fly 3000 miles at a speed of 425 mph?
- 5) Ed biked the first 30 miles of his bike trip in 2 hours and the remaining 40 miles in 3 hours. What was his average speed for the whole trip?

- 6) Measurement Review
 - a) 32 m = _____ cm
 - b) 0.07 ml = _____ l
 - c) 0.4g = _____ mg
 - d) 72 cm = _____ km
 - e) 320,000 oz = _____ ton
 - f) 3 pt = _____ fl.oz.
 - g) 3½ gal = _____ qt
 - h) 720 in = _____ yd

Proportion Problems.

A new way to solve ratio problems.

Example: Find X given that the two triangles below are similar.



Solution: We set up a proportion in terms of long:short = long:short.

$$x:3 = 9:5 \rightarrow \frac{x}{3} = \frac{9}{5}$$

By *moving along diagonals* we get:

$$X = \frac{3 \cdot 9}{5} \text{ giving an answer of } 5\frac{2}{5}.$$

Example: A recipe calls for 6 cups of flour and 4 cups of water. How much water is needed if the recipe is expanded and 10 cups of flour are used?

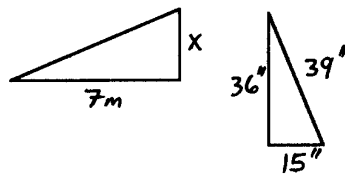
Solution: We set up a proportion in terms of flour:water = flour:water.

$$6:4 = 10:x \rightarrow \frac{6}{4} = \frac{10}{x}$$

By *moving along diagonals* we get:

$$X = \frac{10 \cdot 4}{6} \text{ for an answer of } 6\frac{2}{3} \text{ cups.}$$

- 7) Find X given that these two triangles are similar.



- 8) If $2\frac{2}{3}$ cups of rice require 6 cups of water, then how much water is needed for 5 cups of rice?

- 9) If it takes 39 gallons of gas to drive 715 miles, then how many gallons are needed to drive 400 miles?

- 10) In Harry's class there are 18 boys and 24 girls.

a) What is the ratio of boys to girls?

b) Mary's class has the same ratio of boys to girls. If there are 15 boys, how many girls are there?

- 11) The ratio of cows to goats on a farm is 5:2. If the combined number of goats and cows is 119, then how many goats are there?

- 12) The below map has a (verbal) scale of 1 in = 300 miles. What is the distance, in reality, between the two locations marked on this map?



- 13) The true distance between two cities is 450 miles, but on a map it measures 3 inches. What is the (verbal) scale of the map?

Proportions & Dimensional Analysis – Group Sheet #1

Two Methods for doing Unit Conversion Problems

Example: How many cups are in 5.2 liters?

Solution using The Intuitive Approach:

Since there is nothing on the *Conversion Table* that tells us how to go directly from liters to cups, we must do the problem in two steps. One possibility is to first convert from liters to fluid ounces, and then from fluid ounces to cups. In converting from liters to fluid ounces, we know that one liter is about 33.8 fluid ounces. We then ask ourselves whether we should multiply 5.2 by 33.8, or divide 5.2 by 33.8. Only multiplying gives a reasonable answer. Therefore, 5.2ℓ is $5.2 \cdot 33.8 = 175.76$ fl.oz.

Now, in converting to cups, we know that one cup is 8 fluid ounces. We ask ourselves whether to multiply 175.76 by 8 or divide 175.76 by 8. Only dividing gives a reasonable answer. Therefore, our final answer is: $175.76 \div 8 = \underline{21.97}$ cups.

Solution using The Chain Method:

Usually, we only need to use the *Chain Method* for more complicated problems, or if we are really stuck. This method focuses on getting units to cancel until only the desired unit is left.

Mathematically speaking, we are multiplying our original amount by fractions that are equal to one – in other words, where the numerators and denominators are equal. The work looks like this:

$$\frac{5.2 \ell}{1} \cdot \frac{33.8 \text{ fl oz}}{1 \ell} \cdot \frac{1 \text{ cup}}{8 \text{ fl oz}}$$

Notice that all the units cross cancel, except for "cup". On our calculator we do $5.2 \cdot 33.8 \div 8$, giving an answer of 21.97 cups. Also, *the second two fractions have a value equal to one*, because the denominators are equal to the numerators.

1) Conversion problems.

- 29 lb \approx _____ kg
- 300 mL \approx _____ fl.oz.
- 18 mi \approx _____ km
- 9.2 ℓ \approx _____ gal
- 32 in \approx _____ mm
- 0.76 km \approx _____ yd
- 5'8" \approx _____ m
- 1.7 metric tons \approx _____ lb.

2) The length of a running course is carefully measured both as 1250m and 4104ft. *Without using the Conversion Table*, answer the following. Show your work.

- How many feet are in one meter?
- How many meters are in one foot?
- Check your answers for the above questions by looking in the *Conversion Table*. What is the relationship between the two numbers given as answers for parts a) and b)?

3) *Reciprocals*. For each problem, calculate the answer, then afterwards, check your answer (for the harder ones) by looking in the *Conversion Table*.

- 1 yd = 3 ft \rightarrow 1 ft = _____ yd
- 1 gal = 4 qt \rightarrow 1 qt = _____ gal
- 1 cm = 0.01 m \rightarrow 1 m = _____ cm
- 1 mi \approx 1.6093 km \rightarrow 1 km \approx _____ mi
- 1 kg \approx 2.2046 lb \rightarrow 1 lb \approx _____ kg
- 1 ℓ \approx 1.0567 qt \rightarrow 1 qt \approx _____ ℓ
- 1 ft \approx 0.3048 m \rightarrow 1 m \approx _____ ft
- 1 in \approx 2.54 cm \rightarrow 1 cm \approx _____ in

4) Below is a map of Eagle Island. The scale is 1 inch = 2.4 miles. There is a bike path that goes straight across the island, connecting the two furthest apart points.



- How long is the bike path?
- How long would it take (to the nearest minute) to bike across the island if you averaged 13 mph?
- What is the fractional scale of the map?

Percents & Growth Test

Instructions:

- No notes or looking at your homework.
- You may use the table on p65.
- Calculators are allowed, but you must write down whatever goes into your calculator.

1) What is 41% of 500?

2) What is 60 increased by 25%?

3) What is 170% of 300?

4) 850 is what percent of 900?

5) 0.7 is what percent of 830?

6) What is 485 increased by 4%?

7) What is 600 decreased by 15%?

8) What percent increase is it going from 6500 up to 7200?

9) 525 is 28% of what?

10) 837 is 35% more than what?

11) What percent decrease is it going from 85 down to 35?

12) Rob weighs 160 lb., and Jake weighs 140.

a) Rob is what percent of Jake's weight?

b) Jake is what percent of Rob's weight?

c) Rob is what percent heavier than Jake?

d) Jake is what percent lighter than Rob?

13) Pat's house doubled in value over an 8-year period. Give the approximate average annual rate at which the value was increasing during that period?

14) A shirt normally listed for \$32 is on sale for a 35% discount. What is the new discounted price?

15) Jeff paid \$57.24 for a book after tax. What was the price of the book before tax? (The tax rate is 8%)

16) Beth put \$550 into a savings account at 4.2% interest (compounded annually). What will her balance be after 30 years?