Divisibility Rules

- A number is evenly <u>divisible[1] by 2</u> only if it is even.
- A number is evenly <u>divisible by 3</u> only if the sum of the digits is divisible by 3. The nice thing here is that we can *cast out threes* or groups of digits adding to multiples of three (3, 6, 9, 12, etc.). For example, with 65387 we can immediately cast out the 6 and 3 because they are divisible by 3, and then we can cast out the 8 and 7 because they add to 15. This leaves us with just the 5, which is not divisible by 3, so *we conclude that 65387 is not evenly divisible by 3*.
- A number is evenly <u>divisible by 4</u> only if the last two digits are divisible by 4. For example, 6380716 is evenly divisible by 4, because it ends in 16, which is evenly divisible by 4.
- A number is evenly <u>divisible by 5</u> only if the number ends in a 5 or a 0.
- A number is evenly <u>divisible by 9</u> only if the sum of the digits is divisible by 9. Again, we can *cast out nines* in order to check divisibility for 9 quickly. If we cast out nines and are left with nothing in the end, then the number is evenly divisible by nine. For example, for 71,284 we cast out the 7 and 2 and then cast out the 8 and 1 and we are left with just a 4, so the whole number is not evenly divisible by nine.
On the other hand, with 2,381,697 we cast out the 8 and 1, the 6 and 3, the 2 and 7, and the 9, leaving us with nothing. Therefore, we can conclude that 2,381,697 is evenly divisible by nine.
- A number is evenly <u>divisible by 10</u> only if the number ends in a 0.
- A number is evenly <u>divisible by 6</u> only if it is divisible by both 2 and 3.
Example: 577,368 is evenly divisible by 6 because it is divisible by both 2 and 3 .

- A number is evenly <u>divisible by 8</u> only if the last 3 digits are divisible by 8. This is because it will evenly divide into any number of thousands.
Example: 8,736,104 is *not* evenly divisible by 8 because the last three digits aren't divisible by 8.

- A number is evenly <u>divisible by 12</u> only if it is divisible by both 4 and 3.
Example: 57,481,932 is evenly divisible by 12 because it is divisible by both 4 and 3.

- A number is evenly <u>divisible by 11</u> only if the difference of the sums of every other digit is evenly divisible by 11.

Example: With 6,273,905, we get one sum by adding the digits 6, 7, 9, and 5 to get 27. The other sum comes from adding the digits 2, 3, and 0, which gives 5. The difference of the two sums is 27−5, which is 22. And since 22 is evenly divisible by 11, then we can say that the original number 6273905 is also evenly divisible by 11.

Example: With 378,543 both sums are equal to 15, making the difference equal to zero. Since zero is evenly divisible by 11, then we can say that 378543 is also evenly divisible by 11.

Example: With 68,479, the two sums are 19 and 15, which have a difference of 4. Therefore, we conclude that 68479 is not evenly divisible by 11.

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[1] "Evenly divisible" means it can be divided with no remainder.