8th Grade Assignment – Week #8

Extra Lecture: See if you can watch this lecture on conic sections before Tuesday's group meeting.

A Final Video: After you have watched all the lectures for week #8 and completed all of your work, you should watch this video about the construction of the world's only rhombic triacontahedron house.

Individual Homework: Do **Pythagorean Theorem – Practice Sheet #3**.

Group Assignment: for Tuesday or Thursday

- The *Conic Sections* (as explained in the above lecture) are the results of slicing through a cone. There are three variations: ellipse (which could be a circle), parabola, and hyperbola.
- *Share your drawings*. For your conic section drawings of the parabola, ellipse, and hyperbola, you all had your "tree" (directrix) at a different distance from the "fence" (focus). Share your drawings with each other so that you can see what happens to the curve when the distance from the tree to the fence increase or decreases.
- *Cassini Curve*. **You should read the below instructions for the Cassini curve before arriving at your group meeting.** In your group, you need to decide which size paper to have, and then decide what the distance between the focal points (the two trees) each person should be assigned to. Then help each other out with the drawing, as described below.
- *Puzzle!* Slicing a Triangle
  Show how an equilateral triangle can be divided into
  4 congruent triangles. Now show how an equilateral triangle can be divided into 6 congruent triangles. What other number of congruent pieces can an equilateral triangle be divided into?

Main Lesson Work (geometry) **Coming out of Lecture #1 and Lecture #2**

- *Clay work*
  - Work on a model that is challenge for you, with the intention of saving (and letting dry) a model that you are proud of. Perhaps your goal is to make your best possible dodecahedron – and that is challenging enough. Or, if you have a talent for working with clay, you can try some of the more challenging models that I showed in today's lecture.
- *Paper work – Final Project!!*.
  - Choose one model to be your final project. You may choose either one of Archimedean solids (there are 13 of these), or one of the Archimedean duals (there are also 13 of these). Choose a model that is the right level of difficulty for you.
  - The below pages will help you to make your model. It should be your best work yet!

- *Loci Main Lesson Book Drawing*. **A Cassini Curve**
  Instructions:
  - As explained in the lecture…
    A Cassini curve is the locus of points such that the product of the distances to the two focal points is equal to a constant.
    In other words, the distance to point A times the distance to point B must be equal to the assigned constant product. For us, the assigned constant product will either be 64 or 100.
    We will call this assigned constant, "C".
  - *Your Value for C*. Your first decision is whether to use small paper (8½ by 11 inches), or large paper (around 11 by 14 inches).
    If you use small paper, your C value will be 64.
    If you use large paper, your C value will be 100.
  - *Your Value for f*. Next, you need to decide what the distance should be between the two focal points. **We will call the distance between the focal points, f. Note that all distances are in centimeters.** It is best for everyone in the group to use the same size paper, and to have different values for f.
    - If you are using small paper, the possible values for f should be (in order of importance): 16, 11.3, 17, 8, 15, and 20.
    - If you are using large paper, the possible values for f should be (in order of importance): 20, 14.14, 21, 10, 19, and 25.
  - *Placing the Focal Points* (the two "Trees"). With your paper in landscape orientation, mark in black ink two points that are f centimeters apart. These two points should be centered so that they are halfway up from the bottom of the page, and the same distance from the sides of the page.

- *Constructing a table of possible values.* As I did in the lecture, make a table with the columns labeled A (meaning the distance to focal point "A") and B. Come up with various values for A and B that have a product equal to your C value. For example, if your C value is 100, you could try the distance to A as 13, which results in the distance to B as 7.7 (because 100÷13≈7.7). It may be best to use a calculator for this work.
- *Finding Points on the Curve.*
  - Choose a pair of values from your table (which gives two numbers that multiply to your C value). Setting your compass to the smaller number (of centimeters), draw a circle (lightly in pencil) centered around either focal point.
  - Then use your ruler to measure and determine where on that circle (there are normally two points) the distance to the other focal point is equal to the larger number that you selected from the table. (Note that it is very possible that the two numbers selected from the table do <u>not</u> lead to a solution. If so, then try a different pair of numbers that multiply to C.)
  - Confirm that the location is such that the distance to focal point A times the distance to focal point B is indeed equal to the C value. Mark that point in black ink.
  - Now draw the same size circle around the other focal point in order to find two more points on the curve. Erase all circles and construction lines.
  - Repeat the above process using different numbers, until the curve becomes clear.

- *Locating the Outside of the Curve*. Finding exactly where the curve passing around the outside of the focal involves a fairly advanced calculation. I have done that calculation for you, and you can look up your value with the below table. The value of X tells you how far (in cm) to the outside of the focal point your curve passes.

Table of X values for C=64 (all measurements given in cm)

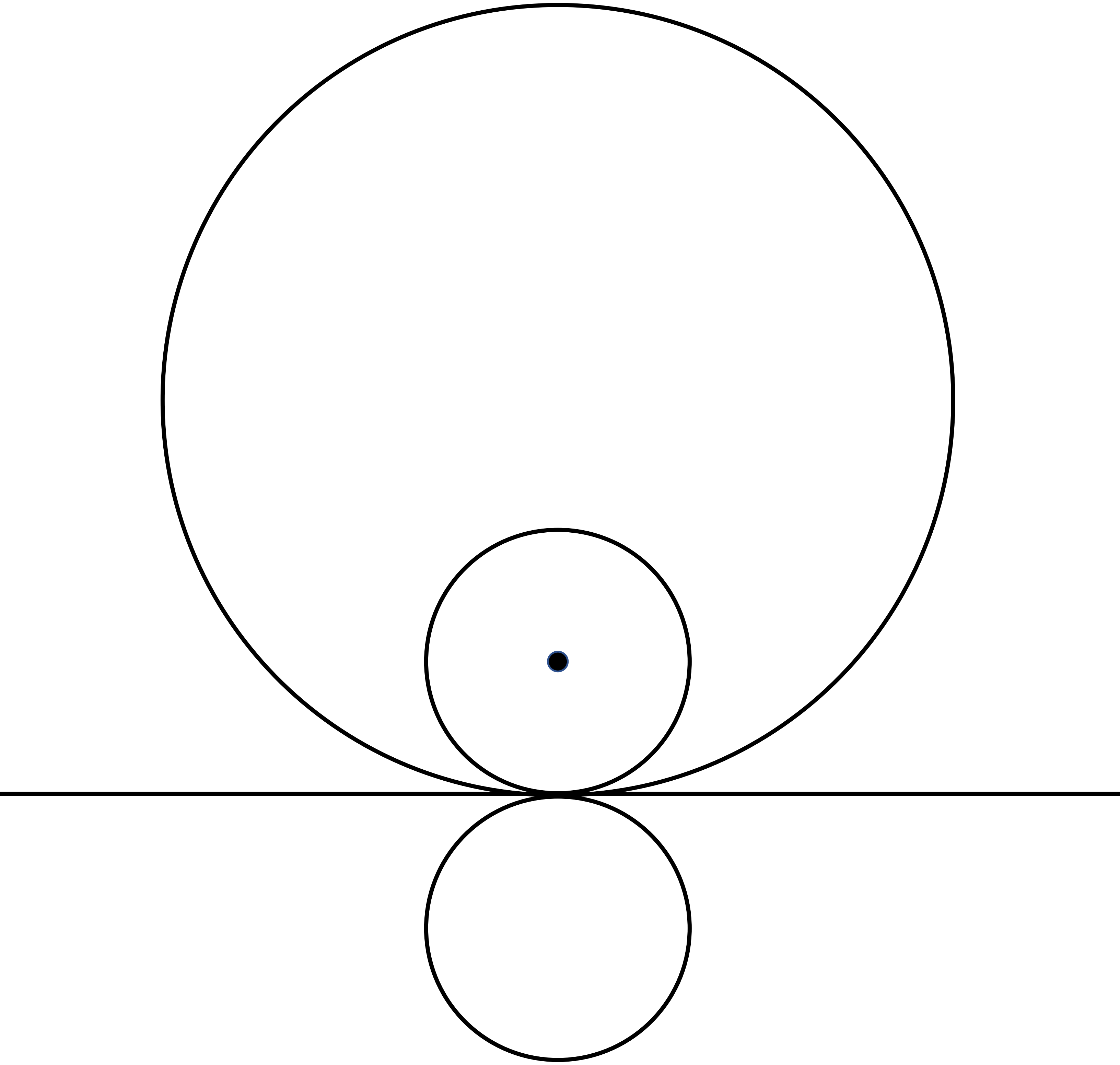
f Value	8	11.3	15	16	17	20
X Value	4.94	4.14	3.47	3.31	3.17	2.81

Table of X values for C=100 (all measurements given in cm)

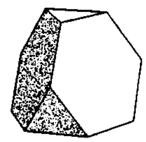
f Value	10	14.14	19	20	21	25
X Value	6.18	5.18	4.29	4.14	4	3.51

- *Drawing the Cassini Curve*. Connect all the points with a smooth curve in colored pencil. Add a title and statement (given above).
- *Share the results* with others in your group!

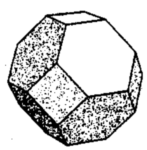
- *Loci Main Lesson Book Drawing.*
  - **Turning the directrix circle inside-out**
    <u>Instructions</u>: Follow the instructions that I gave in the lecture. This should be done freehand. The most important thing is to imagine (as if it were a video) that the directrix circle is growing, which is causing the curve to change, as well. If you wish, you can use the template on the next page to get started. The focal point (tree) and the circles (fences) should be in a cool color (blue), and the curves should be in a warm color (pink, red, or magenta).



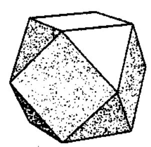
The Archimedean Solids…



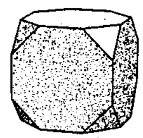
truncated tetrahedron



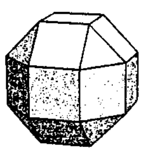
truncated octahedron



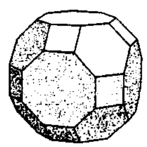
cuboctahedron



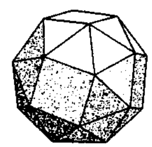
truncated cube



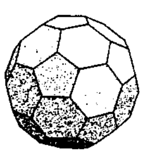
rhombicuboctahedron



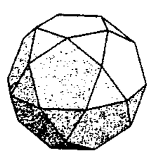
great rhombicuboctahedron



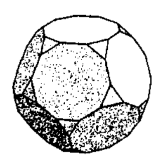
snub cube



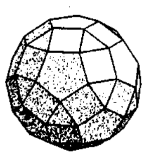
truncated icosahedron



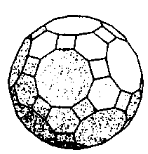
icosidodecahedron



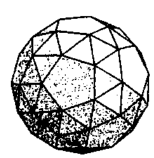
truncated dodecahedron



rhombicosidodecahedron

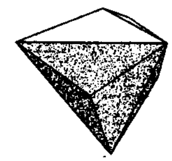


great rhombicosidodecahedron

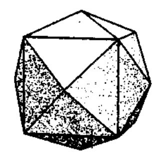


snub dodecahedron

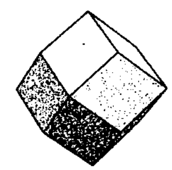
and their Duals



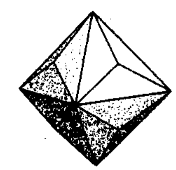
triakistetrahedron



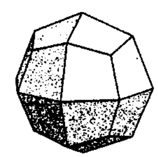
tetrakishexahedron



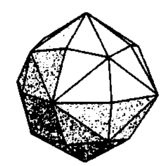
rhombic dodecahedron



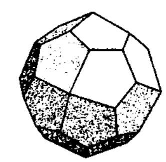
triakisoctahedron



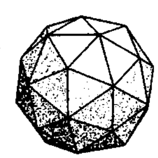
trapezoidal icositetrahedron



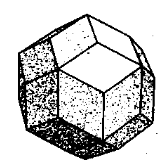
hexakisoctahedron



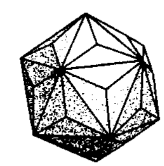
pentagonal icositetrahedron



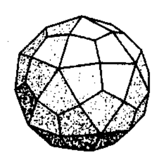
pentakisdodecahedron



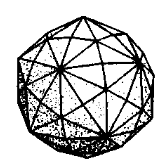
rhombic triacontahedron



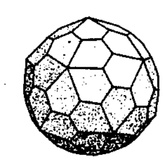
triakisicosahedron



trapezoidal hexecontahedron

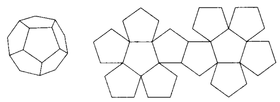


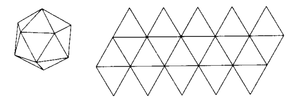
hexakisicosahedron

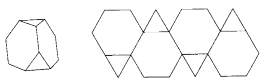


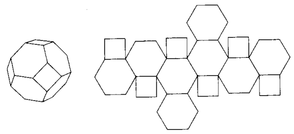
pentagonal hexecontahedron

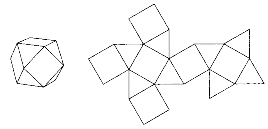
Polyhedron Nets

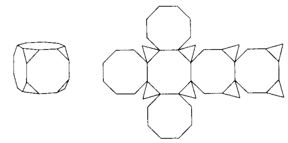

dodecahedron

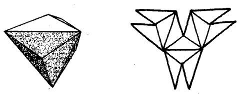

icosahedron

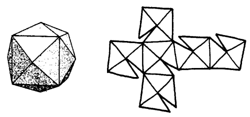

truncated tetrahedron

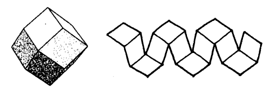

truncated octahedron

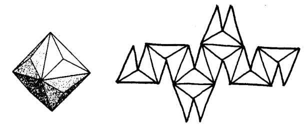

cuboctahedron

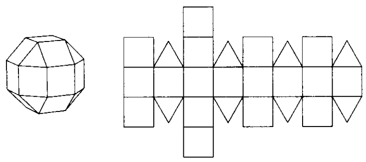

truncated cube

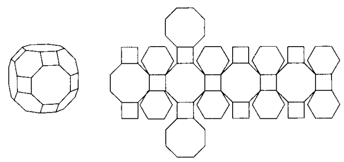

triakistetrahedron

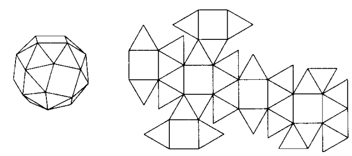

tetrakishexahedron

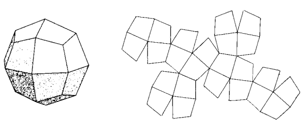

rhombic dodecahedron

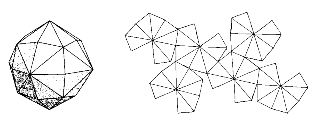

triakisoctahedron

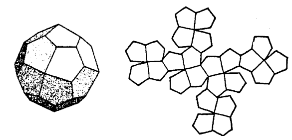

rhombicuboctahedron

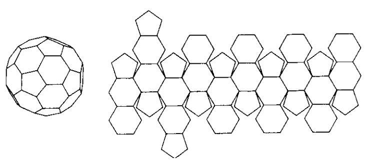

great rhombicuboctahedron

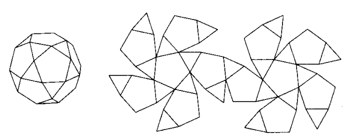

snub cube

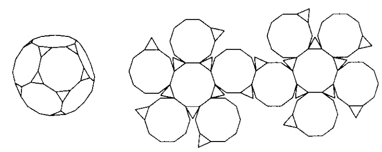

trapezoidal icositetrahedron


hexakisoctahedron


pentagonal icositetrahedron


truncated icosahedron


icosidodecahedron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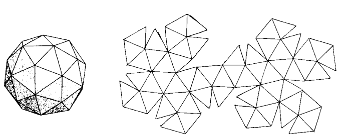
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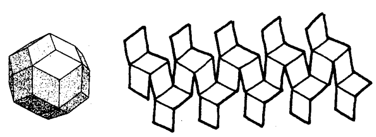
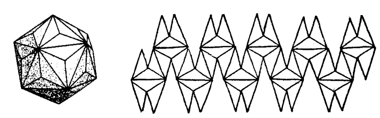
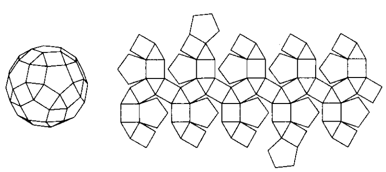
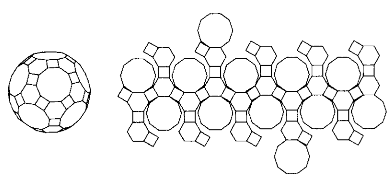
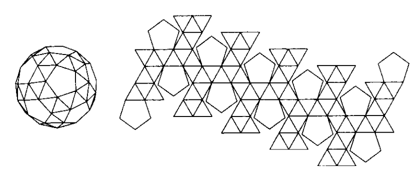
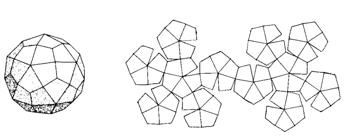

rhombic triacontahedron

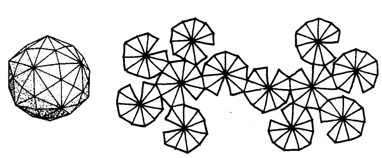

triakisicosahedron

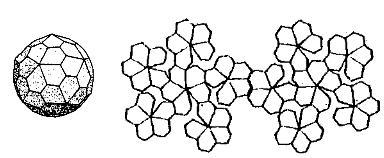

rhombicosidodecahedron


great rhombicosidodecahedron


snub dodecahedron


trapezoidal hexecontahedron

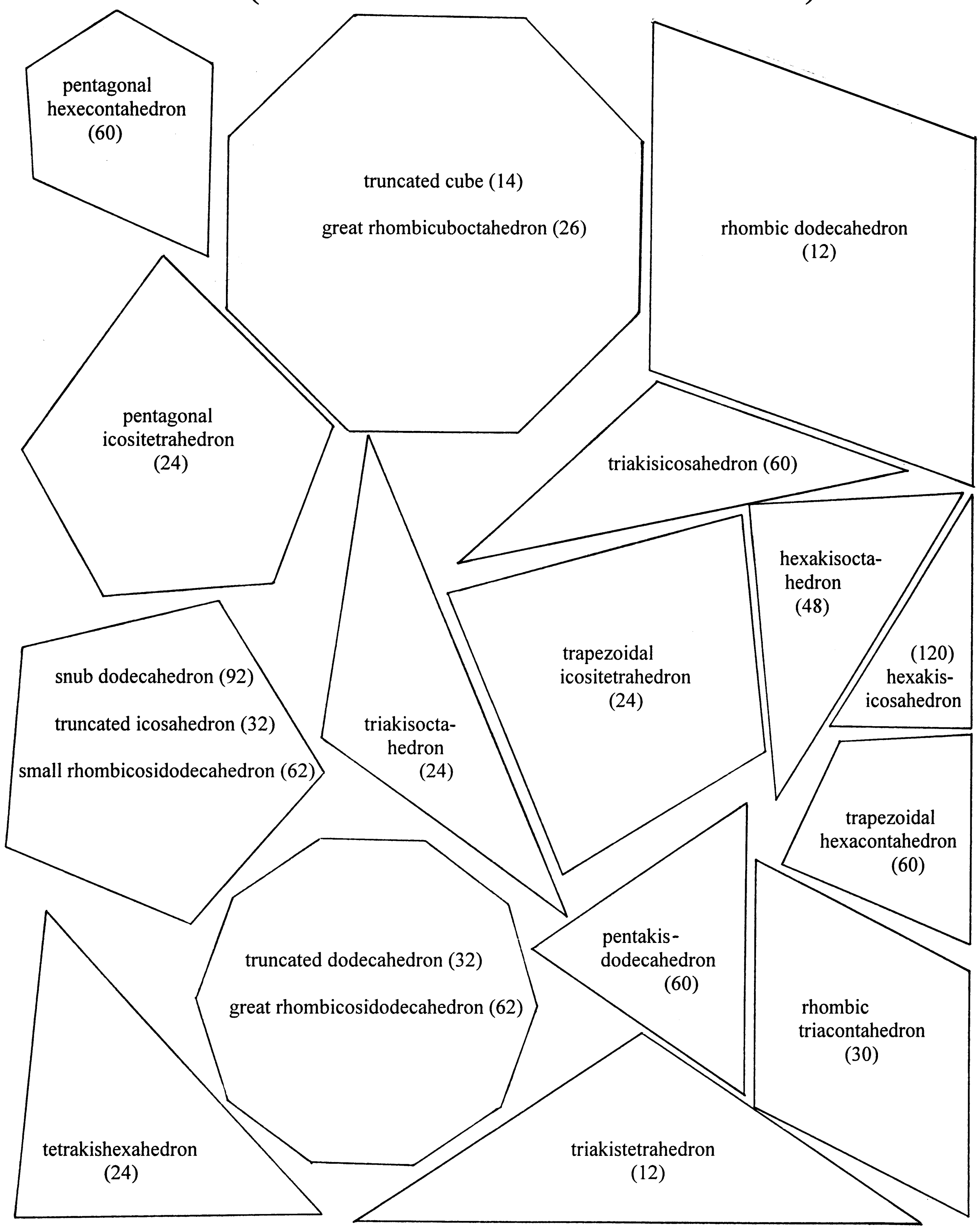

hexakisicosahedron


pentagonal hexecontahedron

Note: Some of the above nets are *not* drawn exactly to scale.

Patterns for the Archimedean Duals

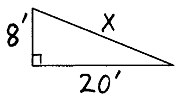
(and some of the Archimedean Solids)

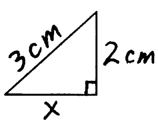


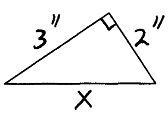
<u>Note</u>: The number in the parenthesis indicates the number of faces on the solid.

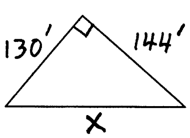
Pythagorean Theorem – Practice Sheet #3

1) Find X, either by using the *Hypotenuse Formula* $c^2 = a^2 + b^2$, or the *Leg Formula* $a^2 = c^2 - b^2$, or Pythagorean triples.

a) 

b) 

c) 

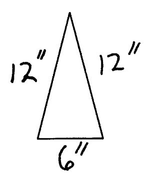
d) 

2) Calculate the exact value. Use the square root algorithm, if necessary.

a) Calculate the length of the diagonal of a rectangle that measures 2m by 1m.

b) Calculate the height of an equilateral triangle with 8-inch long edges.

c) Calculate the height of this triangle.



3) *Challenge!* The body diagonal of a cube is the line that goes from one corner of the cube, through its center, and then to the opposite corner. What is the length of the body diagonal of a cube with one-foot long edges?

4) Below, you are given the length of the three sides of a triangle. State whether the triangle is right, obtuse, or acute.

a) a=12; b=20; c=25

b) a=7; b=2.4; c=7

c) a=7; b=2.4; c=7.4

d) a=7; b=2.4; c=8

e) a=36; b=23; c=40

5) On previous sheets you have calculated the values of the square roots of 2 up through 10. You should now memorize them, since they occur frequently. They are:

$\sqrt{2} \approx 1.414$ $\sqrt{6} \approx 2.45$
$\sqrt{3} \approx 1.73$ $\sqrt{7} \approx 2.65$
$\sqrt{5} \approx 2.24$ $\sqrt{10} \approx 3.16$
$\sqrt{8} \approx 2.83$ $(= 2\sqrt{2})$