8th Grade Assignments – Week #6

Individual Work:

- This week, there is no additional individual work (or work from the workbook) beyond the below main lesson work.

Group Assignment:

For Tuesday

- Work on any of the below items that interest you. It is likely too much to complete during your group work time. Given that there is no individual work from the workbook this week, you can (only if you want to) work individually more on the below problems which weren't completed by your group.
- Select problems from **Pythagorean Theorem – Group Sheet #2**
- Look at the net for a cube shown on the sheet "Tips for Constructing Paper Models" (see below). How many different nets are there for a cube? (Two nets are considered the same if one can be rotated or reflected to produce the other.)

For Thursday

- Perhaps finish anything left from Tuesday's group work.
- Recall that all the points on a parabola are equidistant from a given point (the "tree") and a given line (the "fence"). As I stated in the lecture, your task is to find a method for constructing a parabola using only a compass and a straightedge. In order to simplify the task slightly, we can think of the problem in these terms:

  If I give you a random compass setting, how can you find a location on the page that is exactly that far (the given distance) from both the point and the line?

Main Lesson Work (geometry) (**Coming out of Lecture #1**)

Clay work. Transforming a cube.

Tips for working with clay:

- During this main lesson, I recommend working on clay for about 30 minutes, 2 out of 3 days.
- Make sure the clay is not too moist. You may need to slice it thinly, and let it sit for a couple of hours.
- Use a ball of clay about the size of a baseball.
- Use a sponge to keep your hands moist (but don't use the sponge on the clay itself).
- Always keep the clay rotating. Make a slight improvement, then rotate once every few seconds – always in a random direction. The top of the form should change every time you rotate.
- Make sure the edges don't get too sharp, the points don't get too pointed, and the faces don't get indented.
- The main purpose of this clay work is the process of transformation – therefore, you should only rarely save the form (and let it dry) – maybe just save a couple of models in the last week of the block.
- When you are done with working on clay for the day, you can either preserve it in a plastic bag (with a couple of drops of water) so you can continue working on the same model tomorrow, or tear up the clay into little pieces, sprinkle a few drops of water onto it, and seal it in a plastic bag so you can reuse it and start over next time.

Instructions:

- Work the clay quickly into a sphere by slapping it, then use only your finger tips to form it into a cube.
- Then, as I explained in the lecture, push in the 8 points of the cube. First, the points become small triangles, then the triangles grow until these triangles touch each other point-to-point. Then when they grow further, these triangles transform into hexagons, which somehow grow more and more, and then transform back into (rather larger) triangles. All the while, the original (square) faces keep shrinking, until they shrink to points. See if you can discover the final form!

Paper work.

- Here is a video I created which shows me making a paper model. It will help you to make your own paper models. You should watch it!

Creating a Cube from paper.

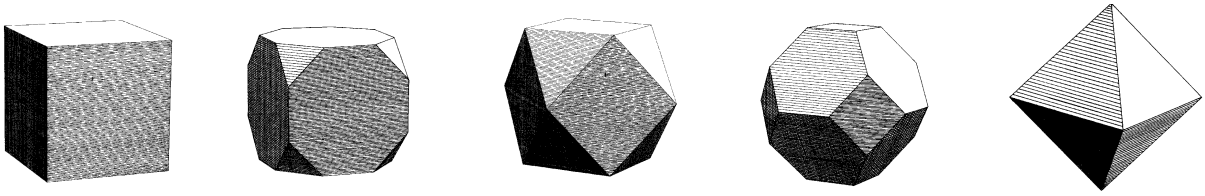
Specifications: From one sheet of paper (if possible), make a cube that has 3-inch (7cm) edges.

Instructions:

- Follow the instructions as given (below) on the sheet "Tips for Constructing Paper Models".
- To give you an idea of how you might color your paper model, here is a photo (above, right) of my Trapezoidal Icositetrahedron (I bet you always wanted one of them, didn't you?) Of course, there are many different techniques and styles you may use to color your model. Also, here's a photo of my stereometry mobile – which has displayed a selection of my children's models in the corner of our living room for many years now.

Main Lesson Work (geometry) (**Coming out of Lecture #2**)

- *Clay work.* Keep working on the transformation of the cube into the octahedron (as I demonstrated in today's lecture). You should be doing this kind of clay work at least every other day. Here is a visual aid:

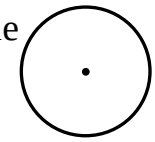


Cube	**Truncated Cube**	**Cuboctahedron**	**Truncated Octahedron**	**Octahedron**
6 squares, 8 pts	6 octagons, 8 Δs	6 squares, 8 Δs	6 squares, 8 hexagons	6 points,8 Δs

- *Paper work.*
  - For each of the three below models, follow the instructions on the next page ("Tips for Constructing Paper Models"). If possible, try to draw the entire net onto one sheet of paper. Make sure each model is done very accurately, and is colored beautifully.
    (1) Cube. (6 squares) Each edge of the cube is 3 inches (or 7.6 cm). Maybe, you did this already.
    (2) Tetrahedron. (4 equilateral triangles) Each edge of the tetrahedron is 4 inches (or 10 cm).
    (3) Octahedron. (8 equilateral triangles) Each edge of the octahedron is 3.5 inches (or 8.9 cm)

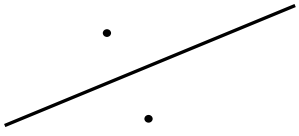
- *Loci Main Lesson Book Page*. (Only to be done once you finish the above work.)

A Circle



The Locus of Points
a Set Distance from a Point

A Perpendicular Bisector



The Locus of Points
Equidistant from Two Points

Tips for Constructing Paper Models

General Tips

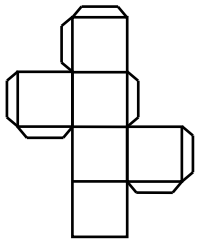
- **Equipment you will need:** Ruler, compass, pencil, scissors, glue, toothpick, pin, colored pencils.
- Constructing these models out of paper is a real exercise in accuracy and careful work. Careless work results in a form that can't come together properly.
- It is important to use good paper[1]. If the paper is too thin, then it won't hold its form; if it's too thick, then it won't fold nicely. Use the paper sparingly.
- It is best to do any artwork on the paper after it has been cut out, and after the folds have been made, but before it has been glued together.

Making the Polygon Form(s)

- Each *polyhedron* paper model has faces with specific polygonal shapes. Your polyhedron may have different faces. You will need to make a nearly perfect *form* of each of these polygonal faces.
- **If you *do* have a photocopy of the form**…The polygon form on photocopied paper is not stiff enough to use for tracing, so you should first make a duplicate of it on thick construction paper. To do this, put the photocopy of the form on top of the sheet of construction paper. Then, using a pin, push through all the vertices of the photocopied form thereby making tiny holes on the construction paper underneath. With a ruler carefully draw the polygon form on the construction paper by connecting the holes that were just made. Carefully cut out this polygon.
- **If you *don't* have a photocopy of the form**…For example, your form may be an equilateral triangle. You will then need to *very* carefully construct the equilateral triangle on construction paper, then carefully cut it out.

Drawing the Net

- **Planning.** Think about how the net can be laid out so that it can be cut out and folded up into the desired polyhedron. For example, in order to make a cube, the square form will have to be traced six times in order to create the full net.
- **Tracing.** Using the polygon form that has just been cut out, create the net by tracing adjacent polygons on the sheet of construction paper. If you have a large piece of construction paper, plan it so that the net is toward the edge of the paper, so you don't waste much paper. Use a very sharp pencil when drawing the net. For best results, leave a pencil's width (0.2mm) of space along the edge between adjacent polygons, in order to account for the fold that will occur along this edge.
- **Placing the Tabs.** Once the net has been made, placement of the tabs needs to be determined. The tabs don't need to be drawn or cut out very neatly, since they won't be seen, but they need to run the length of the whole edge. It is important that no edges are left without a tab, and that no edge has two tabs joining it. Considering all of this, draw all the tabs in the proper places. One possible net (with tabs) for constructing a cube is shown at the right. Can you see how this pattern will fold together, and how all the tabs will connect?



A Cube Net with tabs

Putting it together

- **Folding.** After the net is cut out, folds need to be made along certain edges by placing a ruler along the edge, folding the paper up, and then going over the fold a couple of times with your finger nail.
- **Gluing.** The last part of the construction is gluing it together. This is a slow process, since after gluing a few tabs, it must be allowed to dry somewhat before gluing more tabs. It is best if the tabs are strategically placed in the net in such a way that the last face that gets glued has no tabs on it (this is the bottom square in the above drawing). This allows the last face to be gently pressed into place onto tabs (with glue on them) that are connected to other faces.
- **Artwork.** Make it look beautiful!

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[1] **80-pound cover paper works well. You can order it through a paper supply store in 23" by 35" sheets at a cost of about 65¢ per sheet. Alternatively, I find that standard file folders work fairly well.**

Pythagorean Theorem – Group Sheet #2

1) Use the square root algorithm to calculate each of the following.

a) $\sqrt{3}$

b) $\sqrt{5}$

c) $\sqrt{6}$

d) $\sqrt{2}$ (Round to six significant digits.)

2) Fill in the blanks with *greater than*, or *less than*, or *equal to*.

a) In any *right* triangle, the square of the hypotenuse is ___ the sum of the squares of the other two sides.

b) In any *obtuse* triangle, the square of the hypotenuse is ___ the sum of the squares of the other two sides.

c) In any *acute* triangle, the square of the hypotenuse is ___ the sum of the squares of the other two sides.

3) Below, you are given the length of the three sides of a triangle. State whether the triangle is right, obtuse, or acute. (You may use the *Table of Squares*.)

a) a = 15; b = 20; c = 20

b) a = 15; b = 20; c = 25

c) a = 15; b = 20; c = 30

d) a = 28; b = 96; c = 100

e) a = 7; b = 8; c = 11

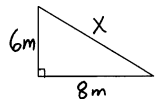
f) a = 5; b = 6; c = 7

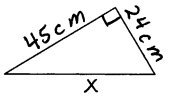
g) a = 3; b = 4; c = 5

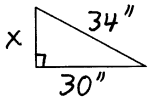
h) a = 2; b = 3; c = 4

4) A ***Pythagorean triple*** is three whole numbers that can be the lengths of the three sides of a right triangle. All of the *primitive* (i.e., reduced) Pythagorean triples that are less than 100 are listed at the back of the book next to the *Table of Squares*.

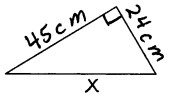
Use Pythagorean triples to determine the length of x.

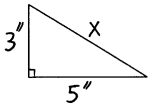
a) 

b) 

c) 

5) Use the formula $c^2 = a^2 + b^2$ in order to find X.

a) 

b) 

c)