8^{th} Grade Assignment – Week #5

Individual Work:

- **Number Bases Test.** Things to keep in mind regarding the test:
  - The test is found at the end of this document.
  - The parent should give the test to the student – to be taken by the end of this week.
  - The student should not use notes or the workbook during the test.
  - The student should not see the test until he/she is ready to take it.
  - You should try to take the test within one hour, if possible.
  - After the student has completed the test, the parent should send a photo of the test to the tutor. The tutor should then indicate which problems are not correct, and then the student should correct those mistakes and send it back to the tutor.
  - Calculators should not be used, but the **tables** (for number base multiplication) at the back of the workbook are permitted. (These tables were also included on the assignment page.)
  - The problems on the front side are worth twice as much as the problems on the back.

- **Important!** This needs to be done before your Tuesday Group Meeting
  Do **Pythagorean Theorem – Group Sheet #1**, problem #2. You need a compass and straightedge (or ruler). If you don't have a protractor, you can make a good estimate of the size of the angle.
- See how far you can get with **Pythagorean Theorem – Practice Sheet #1**

Group Assignment for either Tuesday or Thursday

For Tuesday

- **Pythagorean Theorem – Group Sheet #1**. Do the problems in this order:
  - #3 (assuming you first did the above individual work, #2).
  - #1a
  - #4a
  - If you have more time, then do as much of the rest of the sheet as you can.

For Thursday

- Do the *Mystery Computer Program* (which is found on the next page). Your job is to pretend that you are a computer and simply following the instructions one step at a time. The first line of the program is the input, where N is assigned to 500. If you want a bigger challenge, then you can have N = 1000. Once you are finished with the program, you can then ask yourself (because you are a human being, not a machine): What was the purpose of this program?

A Mystery Computer Program

1. Let N = 500 (or 1000 if you want more of a challenge).
2. Find the square root of N. This number without the decimal places is M. (e.g., If N is 500, then M is 22. If N is 1000, then M is 31.)
3. Write down 2 and the odd numbers up to N in a grid. (To save time, the grid is given below. Cross out all the numbers that are larger than N, if N is less than 1000.) Circle 2, which is the first number in the grid.
4. B is the first non-circled, non-crossed-out number. If B is greater than M, then goto step 9.
5. Circle B.
6. <u>If B is less than 12</u>, then cross out multiples of B, starting at B^2 and continuing until you have gone past N. Look for patterns! (This step saves *us* time compared with step 7, but is tough for computers. Why?)
7. <u>If B is greater than 12</u>, then multiply B by all non-crossed numbers starting with B itself (giving B^2) and working up. Cross out each product that you find. (Note: This step needs adjustment if $N \geq 13^3$, which is 2197. This is because this algorithm is not designed to cross out cubes, or larger powers, of primes.)
8. Go to step 4.
9. Circle all non-crossed-out numbers. The numbers that are circled are the prime numbers.

2	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39
41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79
81	83	85	87	89	91	93	95	97	99	101	103	105	107	109	111	113	115	117	119
121	123	125	127	129	131	133	135	137	139	141	143	145	147	149	151	153	155	157	159
161	163	165	167	169	171	173	175	177	179	181	183	185	187	189	191	193	195	197	199
201	203	205	207	209	211	213	215	217	219	221	223	225	227	229	231	233	235	237	239
241	243	245	247	249	251	253	255	257	259	261	263	265	267	269	271	273	275	277	279
281	283	285	287	289	291	293	295	297	299	301	303	305	307	309	311	313	315	317	319
321	323	325	327	329	331	333	335	337	339	341	343	345	347	349	351	353	355	357	359
361	363	365	367	369	371	373	375	377	379	381	383	385	387	389	391	393	395	397	399
401	403	405	407	409	411	413	415	417	419	421	423	425	427	429	431	433	435	437	439
441	443	445	447	449	451	453	455	457	459	461	463	465	467	469	471	473	475	477	479
481	483	485	487	489	491	493	495	497	499	501	503	505	507	509	511	513	515	517	519
521	523	525	527	529	531	533	535	537	539	541	543	545	547	549	551	553	555	557	559
561	563	565	567	569	571	573	575	577	579	581	583	585	587	589	591	593	595	597	599
601	603	605	607	609	611	613	615	617	619	621	623	625	627	629	631	633	635	637	639
641	643	645	647	649	651	653	655	657	659	661	663	665	667	669	671	673	675	677	679
681	683	685	687	689	691	693	695	697	699	701	703	705	707	709	711	713	715	717	719
721	723	725	727	729	731	733	735	737	739	741	743	745	747	749	751	753	755	757	759
761	763	765	767	769	771	773	775	777	779	781	783	785	787	789	791	793	795	797	799
801	803	805	807	809	811	813	815	817	819	821	823	825	827	829	831	833	835	837	839
841	843	845	847	849	851	853	855	857	859	861	863	865	867	869	871	873	875	877	879
881	883	885	887	889	891	893	895	897	899	901	903	905	907	909	911	913	915	917	919
921	923	925	927	929	931	933	935	937	939	941	943	945	947	949	951	953	955	957	959
961	963	965	967	969	971	973	975	977	979	981	983	985	987	989	991	993	995	997	999

The Square Root Algorithm (without zeroes)

(Written in the style of a computer program. For Eighth grade.)

Note: As you follow the algorithm below you will need to carefully keep track of the following variables:
R, X, Y, *Difference, Sum, Product*

1. Enclose the number in a "house" as you would enclose a long division problem. Starting at the decimal point, and working out in both directions, draw short vertical lines that separate the number into pairs of two digits. Make sure that there are at least as many digit-pairs after the decimal place as the number of decimal places that are needed in the answer. Add ending zeroes, if needed. (e.g., In order to calculate $\sqrt{45}$ to three decimal, we would need to add three pairs of ending zeroes and do $\sqrt{45.\ 00\ 00\ 00}$.)
2. Let R be equal to the left-most digit-pair (which may be a single digit) that is inside the "house". Circle it. Draw a small box, large enough to hold one digit, well to the left of R.
3. Let X be a single digit (somewhere from 0 to 9), such that it is as large as possible, but X^2 is still less than or equal to R. Write X both in the box, and immediately below the box.
4. Underneath the digit that is below the box, write down the *Sum* of X plus X. Write the result of squaring X below R, and below that, write the *Difference* of R minus the square of X.
5. If there are no more digit pairs to bring down, then goto step 11.
6. Bring down the next digit-pair, combining it with, and writing it next to, the *Difference* (that was just found). This now forms the new value for R. Circle it.
7. Draw a small box to the right of the *Sum*. If the digit-pair just brought down is the first one after the decimal place, then write a decimal point above this box.
8. We must now choose a special single digit (somewhere between 0 and 9) that will be written both in the box and directly below the box. This special digit below the box will be called Y, and the new value for X will be the result of taking the *Sum* (found to the left of the box), and attaching to the end of it, the special digit in the box. (This means that Y will be equal to the last digit of the new value for X.) This special digit is chosen such that the result of X times Y is as large as possible, but still less than or equal to R. Write the correct choice for this special digit both in the box and below the box.
9. Underneath R, write the *Product* of X times Y, and then subtract it from R, writing this new *Difference* underneath it all.
10. Underneath X and Y, write the *Sum* of X plus Y. Goto step 5.
11. The answer to the square root problem is found by reading the digits in the boxes from top to bottom, with the decimal point possibly in the middle. If the *Difference* is zero, then the answer is exact; otherwise it is an approximation.

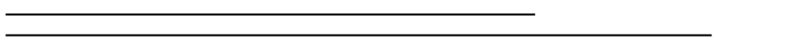
Pythagorean Theorem – Group Sheet #1

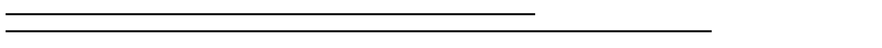
Note: Throughout this whole unit, when calculating the decimal approximation for a square root, answers should be rounded to three significant digits, unless stated otherwise.

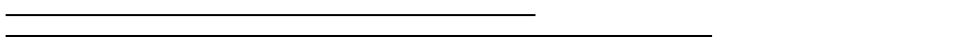
1) Use the square root algorithm to calculate each of the following.

a) $\sqrt{613089}$ b) $\sqrt{1004.89}$ c) $\sqrt{71}$

2) For each problem below, use a compass and a straight edge to construct a triangle (on a separate piece of paper) that has sides equal to the three given line segments. Then, use a protractor to find the measure of the resulting triangle's three angles.

a) 

b) 

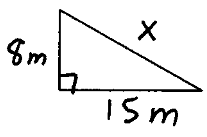
c) 

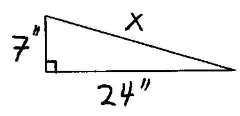
3) Notice that for the above problem, each of the three triangles has the same length for their shortest side and the same length for their middle-sized side.

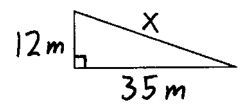
a) Describe how the angles change as the longest side gets longer (until no triangle can be formed)?

b) The Pythagorean Theorem says something about one of the three triangles. Which triangle is it, and what does it say?

4) With each triangle, Calculate the length of the missing side, X.

a) 

b) 

c) 

Pythagorean Theorem – Practice Sheet #1

1) Use the square root algorithm to calculate each of the following. (Each answer is a whole number.)

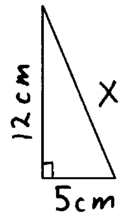
a) $\sqrt{5776}$

b) $\sqrt{222784}$

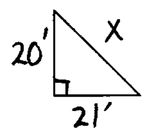
c) $\sqrt{6568969}$

2) Calculate the length of X.

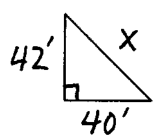
a)



b)



c)



3) State what the Pythagorean Theorem says and what it means.

4) *Challenge!* Calculate $\sqrt{21}$, rounding your answer to six significant digits.

Number Bases – Test Name:__________ Date:________

1) Convert each octal number into decimal:

a) 72_{oct}

b) 254_{oct}

c) 15364_{oct}

2) Convert each decimal number into octal:

a) 18_{dec}

b) 515_{dec}

c) 1600_{dec}

3) Write down the next four numbers past each given number:

a) 25_{oct}

b) 7376_{oct}

4) Convert each number into *standard decimal form*:

a) $6.32 \cdot 10^{7}$

b) $5.2 \cdot 10^{-3}$

5) Convert each number into *scientific notation*:

a) 97,000,000

b) 0.00003283

Do the indicated arithmetic.

6) $363_{oct} + 532_{oct}$

7) $74_{oct} - 46_{oct}$

8) $64_{oct} \times 37_{oct}$

9) $536_{oct} \times 76_{oct}$

10) Convert:

a) 1234_{five} into decimal

b) $1A8_{hex}$ into decimal

11) Convert:

a) 35_{dec} into binary.

b) 730_{dec} into hexadecimal.

12) Write down the next four numbers past each given number:

a) 10110_{bin}

b) $F9E_{hex}$

Do the indicated arithmetic.

13) $\begin{array}{r} 3204_{five} \\ \underline{+4322}_{five} \end{array}$

14) $\begin{array}{r} 10101_{bin} \\ \underline{+1011}_{bin} \end{array}$

15) $\begin{array}{r} D57_{hex} \\ \underline{-EA}_{hex} \end{array}$

16) $\begin{array}{r} 5C_{hex} \\ \underline{x3B}_{hex} \end{array}$

17) *Challenge!* Leave your answer as a repeating decimal: $4F4A_{hex} \div 5B_{hex}$