Problem Set #4

The Number e

On the last problem set we calculated the interest earned in an account that had *continuous compounding*.

For each problem calculate the balance in an account with continuous compounding given $\mathbf{P_0}$ (the initial deposit), **r** (the APR), and **t** (the number of years). Think about how you did it on the last problem set.

1) P_0 = $7000, r = 5%, t = 4 yrs
2) P_0 = $1, r = 100%, t = 1 year

This last problem may have seemed a bit odd, but it turns out to be very useful. In reality, to get it you had to take the *Exponential Growth Formula* and after putting in a value of 1 for P_0, r, and t, we got:

$$P = 1(1+1)^1.$$

But then we adjusted it in order to do multiple compoundings in a year, giving us:

$$P = \left(1+\frac{1}{n}\right)^n$$

where n is the number of compoundings per year

Putting 12 in for n would give us the balance (for P_0 = $1, r = 100%, t = 1 year) for monthly compounding. Simply allowing n to become larger and larger gives us a special value – the number e.

The number e is then defined as:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

This special mathematical identity is read, in the language of calculus, as, "e is equal to the limit, as n approaches infinity, of $\left(1+\frac{1}{n}\right)^n$"

3) Calculate e to ten significant digits.
4) We will now *derive the Continuous Growth Formula* (for interest compounded continuously). To do this, we will combine the *Compound Interest Formula* with the formula for e (above).

   First, we will change the *Compound Interest Formula* such that inside the parentheses we have

   $\left(1+\frac{1}{Q}\right)$ instead of $\left(1+\frac{r}{n}\right)$,

   where $Q = \frac{n}{r}$

   a) Change the *Compound Interest Formula* so that it uses Q instead of n.
   b) Derive the *Continuous Growth Formula* by allowing Q to approach infinity. (The formula should include e and r, but not n.)
5) Use the *Continuous Growth Formula* (the formula you just derived) to calculate the balance of an account compounded continuously where…
   a) P_0 = $7000, r = 5%, t = 4 yrs
   b) P_0 = $950, r = 2.4%, t = 10 years
   c) P_0 = $39,000, r = 3.8%, t = 12 years