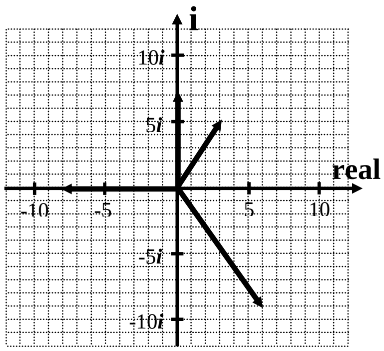
Problem Set #2

The Complex Number Plane

The questions posed at the end of the last problem set are addressed by combining the imaginary number line with the real number line. The result is the *complex plane*. Complex numbers are then graphed as vectors on this complex plane.

For example, the vectors for the numbers –8, 7*i*, 3+5*i*, and 6–9*i* are all shown below.



1) Graph each number as a vector on the complex plane.

a) 8 d) –7–2*i*
b) –3*i* e) 0
c) –4+6*i* f) 10*i*

"Size" of a complex number.

The "size" of the complex number, a + b*i*, is known as its *absolute value*. Absolute value is the same as the length of its vector, which is $\sqrt{a^2+b^2}$.

2) Calculate the *absolute value* of each complex number.

a) 3 – 4*i* d) –6–8*i*
b) 2 + *i* e) 7*i*
c) –7+9*i* f) –10

3) Simplify.

a) $(\sqrt{3}\,i)^4$

b) $\left(\frac{\sqrt{5}}{3} + \frac{1}{2}i\right)^2$

c) $(-\sqrt{3} + i)^3$

d) $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^4$

4) Graph a, b, c on one graph and f, g, h on another graph.

a = 5+3*i* **f** = –3 + 5i
b = 2+7*i* **g** = 8 + 6i
c = **a** + **b** **h** = **f** + **g**

5) What can be said about adding complex numbers?

6) Graph a, b on one graph and f, g on another graph.

a = 5+3*i* **f** = 3 – 2i
b = 2•**a** **g** = 4•**f**

7) What can be said about multiplying a complex number by a real number?

8) Graph a, b on one graph and f, g on another graph.

a = 5+3*i* **f** = –8 + 2i
b = *i*•**a** **g** = *i*•**f**

9) What can be said about multiplying a complex number by *i*?

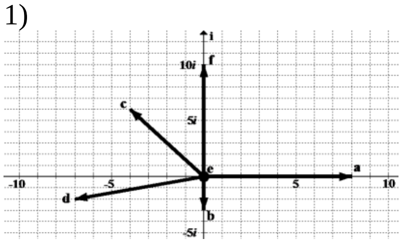
10) Graph a, b, c on one graph and f, g, h on another graph.

a = 1+2*i* **f** = 3 + 4i
b = 2+3*i* **g** = 4 + 3i
c = **a**•**b** **h** = **f**•**g**

11) What can be said about multiplying two complex numbers?

Solutions

1)



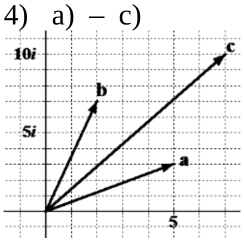
2)

a) 5
b) $\sqrt{5}$
c) $\sqrt{130}$
d) 10
e) 7
f) 10

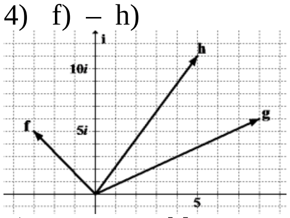
3)

a) 9
b) $\frac{11}{36} + \frac{\sqrt{5}i}{3}$
c) $8i$
d) -1

4) a) – c)

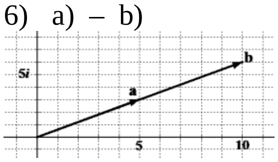


4) f) – h)

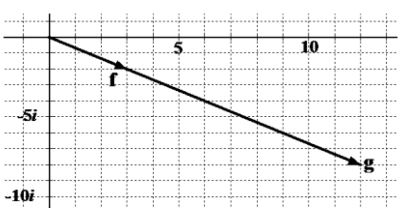


5) One possible answer: The resulting vector is found by placing the two given vectors "tip to tail".

6) a) – b)

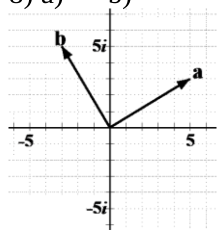


6) f) – g)

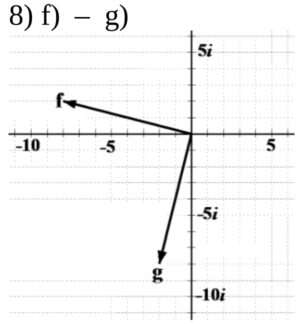


7) The resulting vector maintains the same angle (direction), and its magnitude becomes *n* times greater (where *n* is the real number).

8) a) – b)

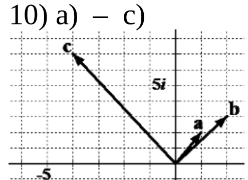


8) f) – g)

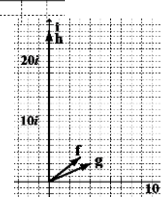


9) Multiplying by *i* simply rotates the vector counterclockwise, by 90°.

10) a) – c)



10) f) – h)



11) The resulting vector has a magnitude equal to the product of the magnitudes of the given vectors, and an angle (formed with the right portion of the real axis) equal to the sum of the two given vectors' angles.