Solutions for Logarithms Practice (for 11th grade workshop)

The Laws of Logarithms

In the Logarithms unit of our *10th Grade Workbook*, the students are led to discover the laws of logarithms for themselves. Here they are:

- $\log_b M{\cdot}N \leftrightarrow \log_b M + \log_b N$
- $\log_b (M/N) \leftrightarrow \log_b M - \log_b N$
- $\log_b N^k \leftrightarrow k \cdot \log_b N$
- $\log_b (1/N) \leftrightarrow -\log_b N$
- $\frac{1}{\log_b a} \leftrightarrow \log_a b$
- $\log_b (b^k) \rightarrow k$
- $b^{\log_b N} \rightarrow N$

(From Logarithms – Part III, Problem Set #1, p51)

Evaluate by using the Properties of Logs.

27) $\log_4(64{\cdot}16) \rightarrow \log_4 64 + \log_4 16 \rightarrow 6+4 \rightarrow$ **10**

28) $\log_5\left(\frac{625}{125}\right) \rightarrow \log_5 625 - \log_5 125 \rightarrow 5-3 \rightarrow$ **2**

29) $\log_8 64^5 \rightarrow 5(\log_8 64) \rightarrow 5(2) \rightarrow$ **10**

30) $\log_3 3^{12} \rightarrow$ 3 to what power is 3^{12}? Answer: **12**

31) $\log_6 6^{14} \rightarrow$ 6 to what power is 6^{14}? Answer: **14**

32) $11^{\log_{11} 8} \rightarrow$ Similarly, log-base-11 and exponent-base-11 are inverse operations, and therefore cancel each other out. Answer: **8**

Still confused?

Try this problem: $2^{\log_2 8}$

(From Logarithms – Part III, Problem Set #4, p55)

7) $\log_4(12x) = 5$

Method #1: change to exponent form
Here's an easier example: $\log_8 64 = 2$
which can be written as $8^2 = 64$
similarly, $\log_4(12x) = 5$ becomes $4^5 = 12x$
$4^5 = 12x \rightarrow 1024 = 12x \rightarrow$ **$x = 256/3$**

Method #2: Exponentiate both sides base-4 (which is the same base as the log). This gives us:

$\log_4(12x) = 5 \rightarrow 4^{\log_4(12x)} = 4^5$

and we know the left side (from the last law of logarithms) is simply 12x, so we have:
$12x = 4^5$, which procedes as before (above).

14b) $15,000 is deposited into a bank account at 3.0092% APR where the interest is compounded quarterly. How long will it take the money to triple.

$45000 = 15000\left(1 + \frac{0.030092}{4}\right)^{4t}$, which is:

$3 = (1.007523)^{4t}$, into log form is:

$\log_{1.007523}(3) = 4t$

$t = \log_{1.007523}(3) \div 4 \approx$ **36.65**