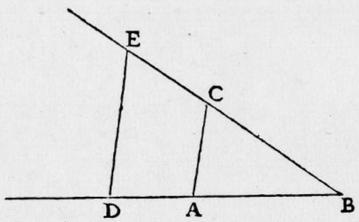
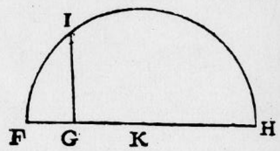
Descartes' Geometric Constructions.
(from "La Géométrie")





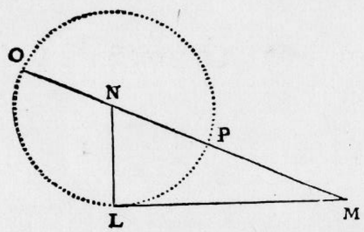
Finding the product of BD and BC.

B

For example, let AB be taken as unity, and let it be required to multiply BD by BC. I have only to join the points A and C, and draw DE parallel to CA; then BE is the product of BD and BC.

If it be required to divide BE by BD, I join E and D, and draw AC parallel to DE; then BC is the result of the division.

If the square root of GH is desired, I add, along the same straight line, FG equal to unity; then, bisecting FH at K, I describe the circle FIH about K as a center, and draw from G a perpendicular and extend it to I, and GI is the required root. I do not speak here of cube root, or other roots, since I shall speak more conveniently of them later.



E

For example, if I have $z^2 = az + b^2$,[21] I construct a right triangle NLM with one side LM, equal to b, the square root of the known quantity b^2, and the other side, LN, equal to $\frac{1}{2}a$, that is, to half the other known quantity which was multiplied by z, which I supposed to be the unknown line. Then prolonging MN, the hypotenuse[22] of this triangle, to O, so that NO is equal to NL, the whole line OM is the required line z. This is expressed in the following way:[23]

$$z = \frac{1}{2}a + \sqrt{\frac{1}{4}a^2 + b^2}.$$

But if I have $y^2 = -ay + b^2$, where y is the quantity whose value is desired, I construct the same right triangle NLM, and on the hypotenuse MN lay off NP equal to NL, and the remainder PM is y, the desired root. Thus I have

$$y = -\frac{1}{2}a + \sqrt{\frac{1}{4}a^2 + b^2}.$$

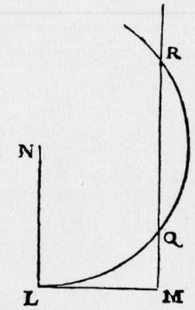
In the same way, if I had

$$x^4 = -ax^2 + b^2,$$

PM would be x^2 and I should have

$$x = \sqrt{-\frac{1}{2}a + \sqrt{\frac{1}{4}a^2 + b^2}},$$

and so for other cases.



E

Finally, if I have $z^2 = az - b^2$, I make NL equal to $\frac{1}{2}a$ and LM equal to b as before; then, instead of joining the points M and N, I draw MQR parallel to LN, and with N as a center describe a circle through L cutting MQR in the points Q and R; then z, the line sought, is either MQ or MR, for in this case it can be expressed in two ways, namely:[24]

$$z = \frac{1}{2}a + \sqrt{\frac{1}{4}a^2 - b^2},$$

and

$$z = \frac{1}{2}a - \sqrt{\frac{1}{4}a^2 - b^2}.$$

And if the circle described about N and passing through L neither cuts nor touches the line MQR, the equation has no root, so that we may say that the construction of the problem is impossible.

Descartes' "Coordinate plane"

G and H