A Summary of **Book I** of Euclid's Elements

<u>**Note:**</u> Some of the theorems have been reworded for clarity.

I-1 *Construction* of an equilateral triangle, given one side.

I-2 *Construction:* To place at a given point (as an extremity) a line[1] equal to a given line.[2]

I-3 *Construction:* Given two unequal lines, to cut off from the greater line a piece equal to the lesser line.[3]

I-4 <u>SAS Congruency Theorem</u>. If the two sides and the included angle of one triangle are congruent to the two sides and the included angle of another triangle, then the third sides are equal, the remaining angles are equal, and the two triangles are congruent.

I-5 <u>Isosceles Triangle Theorem</u>. In an Isosceles triangle, (a) the base angles are equal to one another, and (b) if the two sides are extended, then the angles under the bases will be equal to one another.

I-6 (Converse of #5) If the two base angles of a given triangle are equal, then the sides are also equal.

I-7 Given two lines, drawn from the ends of a third line, and meeting together in a point, there cannot be constructed, on the same third line and on the same side of it, two other lines, with the same lengths and placed on the same extremities as the first two lines, such that they meet at a different point. [This theorem is used to prove I-8.]

I-8 <u>SSS Congruency Theorem</u>. If the three sides of a given triangle are equal, respectively, to the three sides of a second triangle, then all of the angles will be equal, respectively, to one another.

I-9 *Construction:* Bisection of an angle.

I-10 *Construction:* Bisection of a line.

I-11 *Construction* of a line perpendicular to a given line from a point on that line.

I-12 *Construction* of a line perpendicular to a given line from a point NOT on that line.

I-13 <u>Supplementary Angle (Y) Theorem</u>. If two adjacent angles form a straight line, then the sum of the angles is equal to two right angles.

I-14 (Converse of #13) If two adjacent angles add to two right angles, then a straight line is formed.

I-15 <u>Vertical Angle (X) Theorem</u>. Vertical angles are equal.

I-16 In any triangle, the exterior angle is greater than either of the opposite interior angles.

I-17 In any triangle, the sum of any two angles is less than two right angles.

I-18 In any triangle, the greater side subtends the greater angle.

I-19 (Converse of #18) In any triangle, the greater angle is subtended by the greater side.

I-20 In any triangle, any two sides added together are greater than the remaining side.

I-21 If two lines are drawn from the ends of the base of a triangle, such that the point of intersection of the two lines is within the triangle, then the sum of the two lines will be less than the sum of the two sides of the original triangle, and it will contain a greater angle.

I-22 *Construction* of a triangle given three lines.

I-23 *Construction:* Copying a given angle onto a given line.

I-24 If the two sides of one triangle are equal to two sides of a second triangle, then the triangle with the greater included angle has the greater remaining side.

I-25 (Converse of #24) If the two sides of one triangle are equal to the two sides of a second triangle, then the triangle with the greater remaining side has the greater included angle.

I-26 <u>ASA, AAS Congruency Theorem</u>. If two angles and a side of one triangle are equal to two angles and a side of a second triangle, then the triangles are congruent.

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[1] A *line* is what we call today a *line segment*.

[2] With Euclidean constructions, it is technically assumed that whenever a compass is lifted from a page, it collapses, so that it may not be directly used to transfer distances. This theorem allows us to get around this problem, by showing how to construct a circle of a given radius, centered at any given point on the plane.

[3] Essentially, this means to copy a given distance from one line to another, e.g., to measure off AB equal to CD.

I-27 (Converse of #29a) If two lines are cut by a transversal, and alternate interior angles are equal, then the lines are parallel.

I-28 (Converse of #29b and c) If two lines are cut by a transversal, and corresponding angles are equal or the same-side interior angles add to two right angles, then the lines are parallel.

I-29 If two parallel lines are cut by a transversal, then:
- (a) <u>Alternate Interior Angle (Z) Theorem</u> The alternate interior angles are equal.
- (b) <u>Corresponding Angle (F) Theorem</u> The corresponding angles are equal.
- (c) <u>Same-Side Interior Angle (C) Theorem</u> The same-side interior angles add to two right angles.

I-30 Two lines that are parallel to the same line are parallel to each other.

I-31 *Construction* of a line parallel to a given line, through a point not on that line.

I-32 In any triangle:
- (a) <u>Triangle Exterior Angle Theorem</u>. Any exterior angle is equal to the sum of the two opposite interior angles.
- (b) <u>Triangle Interior Angle Theorem</u>. The three interior angles add to two right angles.

I-33 Two lines that join two equal, parallel lines, are themselves parallel and equal.

I-34 In a parallelogram:
- (a) Opposite sides and angles are equal.
- (b) A diagonal divides the parallelogram into two congruent triangles.

I-35 [**]Parallelograms lying on the same base and between the same two parallel lines have equal area.

I-36 [**]Parallelograms having the same length base and lying between the same two parallel lines have equal area.

I-37 [**]Triangles lying on the same base and lying between the same two parallel lines have equal area.

I-38 [**]Triangles having the same length base and lying between the same two parallel lines have equal area.

I-39 Two congruent triangles sharing the same base, and lying on the same side of that base, [but perhaps a mirror image of each other] lie between the same two parallel lines.

I-40 Two congruent triangles having the same length base, and lying on the same side of the same line, lie between the same two parallel lines.

I-41 [**]If a triangle and parallelogram have the same length base and lie between the same two parallel lines, then the area of the parallelogram is twice that of the triangle.

I-42 *Construction* of a parallelogram given an angle and with an area equal to a given triangle.

I-43 If, in any parallelogram, two lines are drawn parallel to two adjacent sides of the parallelogram, such that they intersect on a diagonal of the parallelogram, then the two smaller parallelograms formed on opposite sides of the diagonal have equal area.

I-44 *Construction* of a parallelogram given an angle, a side, and with an area equal to a given triangle.

I-45 *Construction* of a parallelogram given an angle and with an area equal to a given polygon.

I-46 *Construction* of a square with a given side.

I-47 <u>Pythagorean Theorem</u>. In right triangles, [the area of] the square on the side subtending the right angle is equal to [the sum of the areas of] the squares on the other two sides.

I-48 (Converse of #47) In a triangle, if [the area of] the square on one of the sides is equal to [the sum of the areas of] the squares on the remaining two sides of the triangle, then the angle contained by the remaining two sides of the triangle is a right angle.

[**] These are Euclid's "shear and stretch" theorems.