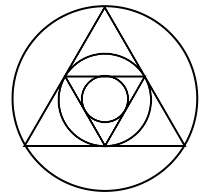
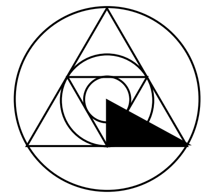
# Mensuration Puzzle Solutions
## for the 10th Grade Online Workshop



1. *Circles and Triangles*

   The Question:

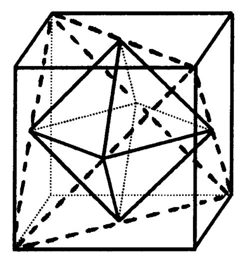
   With the figure shown at the right, what is the ratio of the radii of the largest circle to the smallest circle.

   Solution: Looking at the diagram (shown on the right), we can see that the shortest side of the shaded-in (30-60-90) triangle is the radius of the medium circle, and the longest side of the shaded-in triangle (which is twice the length of the shortest side) is the radius of the largest circle. Therefore, the ratio of the radii of the largest circle to the medium circle is 2:1. Similarly, the ratio of the radii of the medium circle to the smallest circle is 2:1. So, the ratio of the radii of the largest circle to the smallest circle is 4:1.

2. *Nested Polyhedra*

   The Question:

   If you have a cube with a tetrahedron and an octahedron inside it, then what is the ratio of their volumes?

   Solution #1: If we assign the length of the cube's edge equal to 1, then the tetrahedron has an edge of length $\sqrt{2}$, and the octahedron has an edge of length $\sqrt{2}/2$. The formula for the volume of a tetrahedron is $V = (\sqrt{2}/12)\cdot E^3$, and the formula for the volume of an octahedron is $V = (\sqrt{2}/3)\cdot E^3$. (You should derive these two formulas for yourself!) Plugging in the edge lengths into their respective formulas, gives us that the tetrahedron's volume is ⅓, and the octahedron's volume is $^1/_6$.

   Solution #2: Another simpler way to solve the problem is as follows: Picture half the octahedron, which is a square pyramid. The base of the pyramid has an area of ½, and the height of the pyramid is ½. The volume of the pyramid is therefore $^1/_{12}$ and the octahedron is $^1/_6$. For the tetrahedron, picture how it is created by cutting off four of the corners of the cube. Each "cut-off corner" is a (non-regular) tetrahedron itself. It is easiest to imagine that this cut-off tetrahedron has a right triangle as a base (which is half the face of the cube), and it has an edge with a height of 1 that rises perpendicularly and vertically up from the base's right angle. So we can say that the volume of each "cut-off corner" is $^1/_6$. Therefore, the volume of the original desired (regular) tetrahedron is ⅓, which is the volume of the cube (which is 1) minus the volume of the four cut-off corners ($^4/_6$).

   Either way, the ratio of the volumes of the nested polyhedra (as given in the drawing) is <u>Cube : Tetrahedron : Octahedron = 6:2:1</u>.