Basic Euclidean Constructions

Copying a line segment

Instructions: *The intention is to copy line segment AB onto line CD.* Make sure that CD is longer than AB. Set the compass's width equal to AB. Put the needle of the compass on C, and mark an arc that passes through line CD at point E. CE is now equal in length to AB.

Copying an angle

Instructions: *The intention is to copy angle ABC onto line DE.* Set the compass at a width that is a bit less than the shortest of the line segments AB, BC, and DE. Using this width of the compass, draw an arc with the needle at B that passes through both AB (at X) and BC (at W), and then draw an arc with the needle at D that passes through DE at Y. Place the needle at W and adjust the compass so that it reaches to X, and then draw a short arc through X. Keeping this width of the compass, draw an arc, with the needle at Y, that crosses through the previously drawn arc at point Z. Angle ZDY is now equal to angle ABC.

Bisecting a line segment (and construction of the perpendicular bisector)

Instructions: *The intention is to bisect the line segment AB, and to draw a perpendicular bisector* through *it.* Set the compass width so that it is a bit more than half the length of AB. Using this compass width, draw two arcs, one with the needle at A and the other with the needle at B, so that they cross one another in two places – at points C and D. CD is the perpendicular bisector of AB, and M is the midpoint of AB.

Note: This same technique is used to bisect an arc.

Bisecting an angle

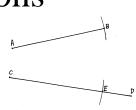
Instructions: *The intention is to bisect angle ABC*. Set the compass width a bit less than both AB and BC. Draw an arc, with the needle at B, that passes through AB at D, and passes through BC at E. Now draw two arcs, both with the same compass width, with the needle at D and then at E, so that the two arcs cross inside angle ABC at point F. The line BF is the desired bisector of the angle ABC. Notice that this will still work if the two arcs are made to intersect "outside" (in this case, to the left of) the angle.

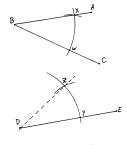
Constructing a perpendicular line through a point on that line

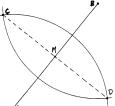
Instructions: The intention is to construct a line perpendicular to AB that passes through X, which is a point on AB. First, draw two arcs, each one using the same compass width and with the needle at X – one arc passing through AX at C and the other passing through XB at D. Now lengthen the compass somewhat and draw two long arcs – one with the needle at C and the other with the needle at D, such that they cross each other at points E and F. Line EF is the desired line; it passes through X and is perpendicular to AB.

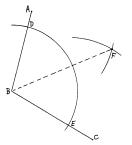
Constructing a perpendicular line through a point not on that line

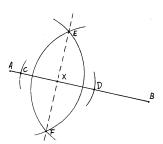
Instructions: The intention is to construct a line perpendicular to AB that passes through X, which is NOT on AB. First, set the compass width a bit longer than the distance that X is from line AB, and then draw an arc, with the needle at X that passes through AB in two points, C and D. Now draw two long arcs using the same compass width, one with the needle at C and the other with the needle at D. They should cross each other at X and at another point E, which is on the other side of AB from X. Line EX is the desired line – it passes through X and is perpendicular to AB.

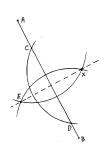












Basic Euclidean Constructions (p2)

Constructing a parallel line

Instructions: *The intention is to construct a line that is parallel to AB and that passes through X, which is not on AB.* Draw line AX significantly beyond X. Set the compass width so that it is a bit shorter than AX, and draw an arc, with the needle at A, that passes through both AB at D and AX at C. Using that same compass width, draw an arc, with the needle at X, that passes through both the extended line AX (at E) and the line (not yet drawn) that passes through X and is parallel to AB. Now, adjusting the compass width, draw a short arc, with the needle at C, that passes through D, and then using the same compass width, draw an arc, with the needle at E that crosses, at point F, the arc that was drawn earlier that passed through E. The line XF is the desired line – it passes through X and is parallel to AB.

Constructing a square, given one side

Instructions: The intention is to construct a square that has each side equal in length to AB. Extend AB past A to N, and then mark point M on AB such that the length of NA is equal to the length of AM. Adjust the compass so that it is somewhat wider than AB and draw two arcs – one with the needle at N, and the other with the needle at M, so that they intersect vertically above A, at point C. Line AC is now perpendicular to AB. Set the compass width equal to AB and draw an arc, with the needle at A, so that it crosses line AC at point D. Using the same compass width, draw two more arcs: one that is horizontally to the right of D, with the needle at D, and a second arc that is above B, with the needle at B. These two arcs cross at point E. Finish the square by connecting the four points ABED.

Constructing a hexagon, inside a given circle

Instructions: *The intention is to construct a regular hexagon inside the given circle.* Draw diameter AB passing through the center of the circle, X. Then set the width of the compass equal to the radius of the circle, and draw one arc with the needle at B, which crosses the circle at points C and D, and another arc, with the needle at A, which crosses the circle at points E and F. The desired hexagon is AFDBCE.

Constructing a pentagon, inside a given circle

Instructions: The intention is to construct a regular pentagon inside the given circle.

- *Method #1*. Draw a diameter and then find the midpoint, Y, of the radius. Point A is located vertically above the center. Placing the needle of the compass at Y, draw an arc through A to point Z on the diameter. The distance A to Z is the desired length of the sides of the pentagon. Copy the length AZ around the perimeter of the circle to get the five points of the pentagon: A, B, C, D, E.
- *Method #2.* Draw horizontal and vertical diameters of the circle. Draw two half-sized circles along the diameter of the original circle. Draw line AX so that it intersects one of the half-sized circles at points B and C. Place the compass needle at point A and draw an arc using AB as the radius, which locates points 3 and 4 on the original circle. Now draw a second arc using AC as the radius (keeping the needle on point A), which locates points 2 and 5 on the original circle. The desired pentagon has points 1, 2, 3, 4, 5 equally spread out on the circle.

