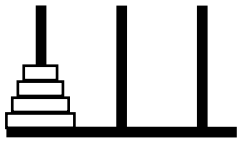
Towers of Hanoi



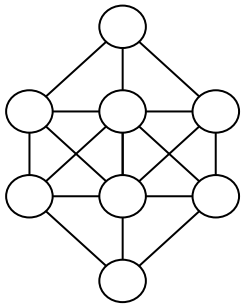
There are three pegs (poles) and a stack of donut-shaped disks placed on one of the pegs. The largest disk is on the bottom of the stack; the smallest is on the top. How many moves does it take to transfer the whole stack to another peg if you can only move one disk at a time, and you cannot place a larger disk on top of a smaller disk? Answer this question first for a stack of 3 disks, then for a stack of 4 disks, then 5 disks, and so on. Finally, answer the classic question: About how long does it take to move a stack of 64 disks to another peg if each move takes 1 second?

Sharing Oranges

Sara has 15 oranges, Bill has 9 oranges, and Stan has none. The oranges are divided equally between the three people, and then Stan is to pay $6 for his share. How can the $6 be divided fairly between Bill and Sara?

Connected Circles

Put the numbers 1 through 8 into the circles such that no two consecutive numbers are connected. For example, if we choose to put 3 into the top-most circle, then we cannot put 2 or 4 into any of the three circles just below it.



Solutions

- Towers of Hanoi

  We make a formula that calculates the number of moves needed (M) based on the number of disks (D). The formula is**:** $M = 2^D - 1$. Therefore, the number of moves needed to move an entire stack of 64 disks is $2^{64} - 1$. This can be estimated by ignoring the minus 1, and realizing that $2^{64} = 2^4 \cdot 2^{10} \cdot 2^{10} \cdot 2^{10} \cdot 2^{10} \cdot 2^{10}$. And since $2^4 = 16$ and $2^{10} = 1024$ (which is approximately 1000) we can say that $2^{64} \approx 16{,}000{,}000{,}000{,}000{,}000$. This is approximately the total number of moves needed to move the whole stack of 64 disks. It is also the total number of seconds needed to move the whole stack, given that each move takes one second.

  How many years is this? We first calculate the number of seconds in a year. There are 60 seconds in a minute, $60 \cdot 60 = 3600$ seconds in an hour, $3600 \cdot 24 = 86400$ seconds in day, and, finally, $86400 \cdot 365 = 31536000$ seconds in a year, which we can approximate as 32,000,000. The number of years is therefore 16,000,000,000,000,000,000 ÷ 32,000,000, which, when looked at as a fraction, reduces nicely to 1,000,000,000,000 ÷ 2, which equals <u>500,000,000,000 years</u>, or half a trillion years. The exact number of years is 584,942,417,335, which shows that our quick estimation is quite accurate.

- **Sharing Oranges**

  <u>Solution</u>: Stan should pay Bill 75¢ and pay Sara $5.25. The reasoning is as follows: Everyone received 8 oranges. Therefore, Bill only gave Stan 1 orange, and Sara gave Stan 7 oranges. Since the ratio of the amount given by Sara and Bill is 7:1 (Sara gave 7 times as much as Stan), they should divide the money in that same ratio. Therefore, Bill gets $^1/_8$ of the money and Sara gets $^7/_8$ of the money (i.e., Sara gets 7 times as much money as Stan).

- **Connected Circles**

  <u>Solution:</u>