Puzzles - for 9th grade workshop

1. Guessing One Number

The teacher says to a group of fifth grade students: "Choose any number, write it down, and circle it. Add 7. Multiply by 3. Subtract the original number. Tell me your final answer." After hearing the student's final answer, the teacher can then determine what the student's original number must have been. Use algebra to determine how the teacher can do this.

2. Two Hourglasses

There are two hourglasses. One runs for 7 minutes, and the other runs for 4 minutes. How can you time a 9-minute interval?

3. Digit Arithmetic Puzzles

With each of the following puzzles, you must put in a digit for each letter. The same digit must be put into the same letters, and different letters must have different digits.

a)
```
  PQ
+ Q
----
  QP
```

b)
```
   ABCD
     CD
+  EFGH
-------
   IJDH
```
At least two solutions are possible!

4. Coin Puzzles

Mark has 30 coins worth $3.90. How many of each type of coin are there if …

a) there are quarters, dimes, and nickels, and there are 50% more nickels than dimes?

b) there are quarters, dimes, and nickels, and the number of nickels is one less than four times the number of dimes?

5. Equal Products

With the configuration on the right, each letter stands for a different (single) digit. Assign values to the letters so that A•B•C and B•D•E and F•E•G are all equal.

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A     F
B  D  E
C     G
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6. Ages of Teenagers

There is a group of teenagers. The product of their ages is 737,100. Find the number of teenagers in the group and the age of each one.

7. Two Pitchers

There are two pitchers – one with a quart of apple juice, and the other with a quart of milk. Mary takes one cup of apple juice from the apple juice pitcher, adds it to the pitcher of milk, and mixes it. Then she pours a cup of the liquid from the mixed pitcher into the pitcher of apple juice. In the end, is there more milk in the apple juice, or more apple juice in the milk?

8. Jill's Bike Ride

Jill went on a bike ride from Brownsville to Manson passing through Gilpin along the way. After 40 minutes, she saw a sign that read: "It is half as far from here to Brownsville as it is from here to Gilpin". 24 miles further along the route, she had finished all but $^1/_5$ of her trip, and it was there that she saw another sign, this time reading: "It's half as far from here to Manson as it is from here to Gilpin". If she biked at the same rate of speed during the whole trip, then how far is it from Brownsville to Manson.

9. Guessing Three Numbers

The teacher says to a group of fifth grade students: "Think of any three single-digit numbers (except zero), in any order. Once you have chosen your three numbers, multiply the first number by two, then add five, and then multiply by five. Now add the second number, subtract four, multiply by ten, add three, and then add the third number. Now tell me your final result."

After hearing the student's final result, the teacher can then determine what the student's original three numbers must have been. Use algebra to determine how the teacher can do this.

10. A Changing Choir

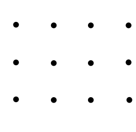
A choir has 29 women and 21 men. How many women need to join the choir so that it becomes 72% women? (Assume that the number of men stay the same.)

11. Stolen Chocolate

Somebody stole Bob's chocolate bar. Jerry, Mary and Larry were suspected. Jerry said, "I didn't do it!" Mary said, "Larry didn't do it!" And Larry said, "Yes, I did!" At least two of them lied. Who stole the chocolate bar?

12. Connect-the-Dot Squares

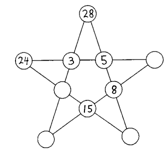
On the four-by-four grid shown on the right, connect four of the dots to make a square. How many possible squares are there?



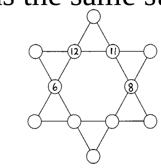
13. Pentagram and Hexagram

Fill in the circles so each row has the same sum.

a)



b)



14. The Snail's Journey

A snail crawled up the outside of a cylindrical water tower, which is 70 feet tall and has a circumference of 24 feet. However, in order to make his journey easier, he crawled up at a slight (but constant) incline such that by the time he made it to the top, he had circled the water tower exactly 7 times. How far did he actually travel?

15. A Long Rope

Imagine that there is a rope going around the equator of the earth. The rope is exactly 1000 feet longer than the equator (which is about 24,880 miles long), and the rope is somehow everywhere held up above the ground by the same height. Could a horse jump over the rope?

Solutions to 9th Grade Puzzles

1. Guessing One Number

We simply translate the teacher's instructions:

> "Choose any number, write it down, and circle it. Add 7. Multiply by 3. Subtract the original number. Tell me your final result."

into the following algebraic expression: $3(x+7) - x$

which simplifies to: $2x + 21$.

In order to get X (the student's original number), we just subtract 21 from the student's final result, and then divide by 2.

2. Two Hourglasses

We first need to "prepare" for the 9-minute interval. To do this we begin by starting both hourglasses at the same time. Once the 4-minute hourglass runs out, we immediately flip it. Once the 7-minute hourglass runs out, we stop the 4-minute hourglass (which has 1-minute's worth of sand left in it) by putting it on its side. We are now prepared for the 9-minute interval. To begin the 9-minute interval we restart the 4-minute hourglass, which will finish in one minute. After it finishes, we complete the 9-minute interval by simply running the 4-minute hourglass two more times. (There are other possible solutions, as well.)

3. Digit Arithmetic Puzzles

a)
```
  89
+  9
----
  98
```

b)
```
  1045   or    1275
    45           75
+ 2867       + 3608
------       ------
  3957         4958
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4. Coin Puzzles

a) 12 nickels, 8 dimes, 10 quarters.

b) 15 nickels, 4 dimes, 11 quarters.

5. Equal Products

Each of the three strings of numbers must be equal, so their prime factorizations must be equal. Therefore, we know that we can't use the digits 0, 5 or 7 because a given digit can only appear in two of the three products. Now let's consider the factor 3. The digit 9 contains two 3's (in its prime factorization) and the digits 3 and 6 each contain one 3. Therefore we will place the 9 at an intersect point, and the 3 and 6 on corners away from the 9. Now let's consider the factor 2.

The digit 8 contains three 2's, the digit 4 contains two 2's, and the digits 2 and 6 each contain one 2. We simply think of this as we fill in the remaining places of the puzzle. The final answer is shown here.

```
8   3
9 2 4
1   6
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6. Ages of Teenagers

There are five teenagers in the group. There are an 18-year-old, a 14-year-old, a 13-year-old, and two 15-year-olds.

7. Two Pitchers

Equal amounts of both! In the end, there is $^4/_5$ of a cup of milk mixed into the apple juice pitcher, and $^4/_5$ of a cup of apple juice mixed into the milk pitcher.

8. Jill's Bike Ride

The distance from the second sign to Manson is $^1/_5$ of the whole distance (Brownsville to Manson). Therefore, the second sign to Gilpin is $^2/_5$ of the whole distance, and the remaining distance (Brownsville to Gilpin) is $^2/_5$ of the whole distance. Since the distance from the first sign to Gilpin is twice as far as the distance from the first sign to Brownsville, the first sign must be ⅓ of the way from Brownsville to Gilpin. And since we just said that Brownsville to Gilpin is $^2/_5$ of the whole trip, we now know that Brownsville to the first sign must be $^2/_{15}$ (= ⅓ • $^2/_5$) of the whole trip, and the distance from the first sign to Gilpin must be $^4/_{15}$ of the whole trip. Therefore, the distance between the two signs must be ⅔ (= $^2/_5$ + $^4/_{15}$) of the distance of the whole trip, or, stated in reverse, the distance of the whole trip must be $^3/_2$ of the distance between the signs, which we know is 24 miles.

So the length of the whole trip is $^3/_2$•24 or <u>36 miles</u>.

Solutions

9. Guessing Three Nu0mbers

Given x, y, and z as the three numbers, the teacher's instructions simply translate to:
$10\,(\,5\,(2x + 5) - 4 + y) + 3 + z.$

The student's final result (R) is then equal to the above expression, which simplifies to:
$R = 100x + 10y + z + 213$

Now we can easily see that the teacher simply takes the student's result (R), and subtracts 213, which leads to the student's three original digits, x, y, and z.

10. A Changing Choir

If the choir is 72% women, then the ratio of men to women must be 28:72, which is 7:18. Since we know the number of men is 21, we can determine the number of women by setting up this proportion: 7:18 = 21:x. Solving this yields x = 54, so 25 new women must join.

11. Stolen Chocolate

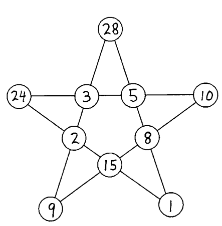
Mary and Larry contradict each other, so one of them must be telling the truth, and the other must be lying. Since we know that at least two are lying, then Jerry must be lying. So Jerry stole the chocolate.

12. Connect-the-Dot Squares

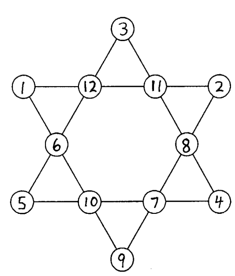
There are 9 squares with sides of length 1, 4 squares with sides of length 2, 1 square with a side of length 3, 4 squares (diamonds) with sides of length $\sqrt{2}$, and 2 (tilted) squares with sides of length $\sqrt{5}$. Therefore the total number of squares is 20.

13. Pentagram and Hexagram

a)



b) One possible solution:



14. The Snail's Journey

We can imagine that we could "unwind" the path he took, and we would get a right triangle where the base would be 168 (which is 7x24), and the height would be 70. The hypotenuse of this triangle is indeed the path he traveled, which turns out to be 182 feet.

15. A Long Rope

At first glance, it may seem that this extra 1000 feet would somehow be spread out across the whole equator, and therefore that the rope would barely be above the ground. But this is not the case. Whether the rope is going around the equator of the earth, the moon, or Jupiter doesn't matter; either way (as long as it is 1000 feet longer than that equator) it will be the same height above the ground.

If we increase the length of a circle's radius by x, then its circumference increases by $2\pi x$. For example, if circle A has a radius of 30m, and circle B has a radius of 40m, then the difference of their circumferences is $2\pi \cdot 10$, which is ≈ 62.8m.

With the problem at hand, we can say that if a circle's circumference increases by 1000, then its radius increases by $1000 \div (2\pi) \approx 159$. Therefore, the fence going around the equator will be about 159 feet high – far too high for any horse to jump over.